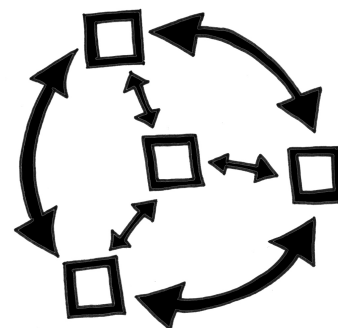# Core Connections, Course 2
Parent Guide with Extra Practice

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Introduction to the Parent Guide with Extra Practice

Welcome to the *Core Connections, Course 2 Parent Guide with Extra Practice*. The purpose of this guide is to assist you should your child need help with homework or the ideas in the course. We believe all students can be successful in mathematics as long as they are willing to work and ask for help when they need it. We encourage you to contact your child's teacher if your student has additional questions that this guide does not answer.

Detailed examples follow a summary of the concept or skill and include complete solutions. The examples are similar to the work your child has done in class. Additional problems, with answers, are provided for your child to try.

There will be some topics that your child understands quickly and some concepts that may take longer to master. The big ideas of the course take time to learn. This means that students are not necessarily expected to master a concept when it is first introduced. When a topic is first introduced in the textbook, there will be several problems to do for practice. Succeeding lessons and homework assignments will continue to practice the concept or skill over weeks and months so that <u>mastery</u> <u>will</u> <u>develop</u> <u>over</u> <u>time</u>.

Practice and discussion are required to understand mathematics. When your child comes to you with a question about a homework problem, often you may simply need to ask your child to read the problem and then ask what the problem is asking. Reading the problem aloud is often more effective than reading it silently. When you are working problems together, have your child talk about the problems. Then have your child practice on his/her own.

Below is a list of additional questions to use when working with your child. These questions do not refer to any particular concept or topic. Some questions may or may not be appropriate for some problems.

- What have you tried? What steps did you take?

- What didn't work? Why didn't it work?

- What have you been doing in class or during this chapter that might be related to this problem?

- What does this word/phrase tell you?

- What do you know about this part of the problem?

- Explain what you know right now.

- What do you need to know to solve the problem?

- How did the members of your study team explain this problem in class?

- What important examples or ideas were highlighted by your teacher?

- Can you draw a diagram or sketch to help you?

- Which words are most important? Why?

- What is your guess/estimate/prediction?

- Is there a simpler, similar problem we can do first?

- How did you organize your information? Do you have a record of your work?

- Have you tried drawing a diagram, making a list, looking for a pattern, etc.?

If your student has made a start at the problem, try these questions.

- What do you think comes next? Why?

- What is still left to be done?

- Is that the only possible answer?

- Is that answer reasonable?

- How could you check your work and your answer?

- How could your method work for other problems?

If you do not seem to be making any progress, you might try these questions.

- Let's look at your notebook, class notes, and Toolkit. Do you have them?

- Were you listening to your team members and teacher in class? What did they say?

- Did you use the class time working on the assignment? Show me what you did.

- Were the other members of your team having difficulty with this as well?
  Can you call your study partner or someone from your study team?

This is certainly not a complete list; you will probably come up with some of your own questions as you work through the problems with your child. Ask any question at all, even if it seems too simple to you.

To be successful in mathematics, students need to develop the ability to reason mathematically. To do so, students need to think about what they already know and then connect this knowledge to the new ideas they are learning. Many students are not used to the idea that what they learned yesterday or last week will be connected to today's lesson. Too often students do not have to do much thinking in school because they are usually just told what to do. When students understand that connecting prior learning to new ideas is a normal part of their education, they will be more successful in this mathematics course (and any other course, for that matter). The student's responsibilities for learning mathematics include the following:

- Actively contributing in whole class and study team and discussions.

- Completing (or at least attempting) all assigned problems and turning in assignments in a timely manner.

- Checking and correcting problems on assignments (usually with their study partner or study team), based on answers and solutions provided in class and online.

- Asking for help when needed from their study partner, study team, and/or teacher.

- Attempting to provide help when asked by other students.

- Taking notes and using their Toolkit when recommended by the teacher or the text.

- Keeping a well-organized notebook.

- Not distracting other students from the opportunity to learn.

Assisting your child to understand and accept these responsibilities will help him or her to be successful in this course, develop mathematical reasoning, and form habits that will help her/him become a life-long learner.

Additional support for students and parents is provided at the CPM website (cpm.org) and at the CPM Homework Help website (homework.cpm.org).

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Outcome: Any possible or actual result of the action considered, such as rolling a 5 on a standard number cube or getting tails when flipping a coin.

Event: A desired (or successful) outcome or group of outcomes from an experiment, such as rolling an even number on a standard number cube.

Sample space: All possible outcomes of a situation. For example, the sample space for flipping a coin is heads and tails; rolling a standard number cube has six possible outcomes (1, 2, 3, 4, 5, and 6).

Probability: The likelihood that an event will occur. Probabilities may be written as fractions, decimals, or percents. An event that is guaranteed to happen has a probability of 1, or 100%. An event that has no chance of happening has a probability of 0, or 0%. Events that "might happen" have probabilities between 0 and 1 or between 0% and 100%. In general, the more likely an event is to happen, the greater its probability.

Experimental probability: The probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

Theoretical probability is a calculated probability based on the possible outcomes when they all have the same chance of occurring.

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes (events)}}{\text{total number of possible outcomes}}$$

In the context of probability, "successful" usually means a desired or specified outcome (event), such as rolling a 2 on a number cube (probability of $\frac{1}{6}$). To calculate the probability of rolling a 2, first figure out how many possible outcomes there are. Since there are six faces on the number cube, the number of possible outcomes is 6. Of the six faces, only one of the faces has a 2 on it. Thus, to find the probability of rolling a 2, you would write:

$$P(2) = \frac{\text{number of ways to roll 2}}{\text{number of possible outcomes}} = \frac{1}{6} \;\text{ or }\; 0.1\overline{6} \text{ or approximately } 16.7\%$$

Example 1

If you roll a fair, 6-sided number cube, what is $P(3)$, that is, the probability that you will roll a 3?

Because the six sides are equally likely to come up, and there is only one 3, $P(3) = \frac{1}{6}$.

Example 2

There are 12 marbles in a bag: 2 clear, 4 green, 5 yellow, and 1 blue. If one marble is chosen randomly from the bag, what is the probability that it will be yellow?

$$P(\text{yellow}) = \frac{5 \text{ (yellow)}}{12 \text{ (outcomes)}} = \frac{5}{12}$$

Example 3

Joe flipped a coin 50 times. When he recorded his tosses, his result was 30 heads and 20 tails. Joe's activity provided data to calculate experimental probability for flipping a coin.

a. What is the theoretical probability of Joe flipping heads?

    The theoretical probability is 50% or $\frac{1}{2}$, because there are only two possibilities (heads and tails), and each is equally likely to occur.

b. What was the experimental probability of flipping a coin and getting heads based on Joe's activity?

    The experimental probability is $\frac{30}{50}$, $\frac{3}{5}$, or 60%. These are the results Joe actually got when he flipped the coin.

Example 4

Decide whether these statements describe theoretical or experimental probabilities.

a. The chance of rolling a 6 on a fair die is $\frac{1}{6}$.

    This probability is theoretical.

b. I rolled the die 12 times and 5 came up three times.

    This probability is experimental.

c. There are 15 marbles in a bag; 5 blue, 6 yellow, and 4 green. The probability of getting a blue marble is $\frac{1}{3}$.

    This probability is theoretical.

d. When Veronika pulled three marbles out of the bag she got 2 yellow and 1 blue, or $\frac{2}{3}$ yellow, $\frac{1}{3}$ blue.

    This probability is experimental.

       Core Connections, Course 2

Problems

1. There are 24 crayons in a box: 5 black, 3 white, 7 red, 2 yellow, 3 blue, and 4 green. What is the probability of randomly choosing a green? Did you respond with an experimental or theoretical probability?

2. A spinner is divided into four equal sections numbered $2, 4, 6$, and 8. What is the probability of spinning an 8?

3. A fair number cube marked $1, 2, 3, 4, 5$, and 6 is rolled. Tyler tossed the cube 40 times, and noted that 26 times an even number showed. What is the experimental probability that an even number will be rolled? What is the theoretical probability?

4. Sara is at a picnic and reaches into an ice chest, without looking, to grab a can of soda. If there are 14 cans of orange, 12 cans of fruit punch, and 10 cans of cola, what is the probability that she takes a can of fruit punch? Did you respond with an experimental probability or a theoretical one?

5. A baseball batting average is the probability a baseball player hits the ball when batting. If a baseball player has a batting average of 266, it means the player's probability of getting of getting a hit is 0.266. Is a batting average an experimental probability or theoretical?

6. In 2011, 39 people died by being struck by lightning, and 241 people were injured. There were 310,000,000 people in the United States. What is the probability of being one of the people struck by lightning?

7. In a medical study, 107 people were given a new vitamin pill. If a participant got sick, they were removed from the study. Ten of the participants caught a common cold, 2 came down with the flu, 18 got sick to their stomach, and 77 never got sick. What was the probability of getting sick if you participated in this study? Did you respond with an experimental probability or a theoretical one?

8. Insurance companies use probabilities to determine the rate they will charge for an insurance policy. In a study of 300 people that had life insurance policies, an insurance company found that 111 people were over 80 years old when they died, 82 people died when they were between 70 and 80 years old, 52 died between 60 and 70 years old, and 55 died when they were younger than 60 years old. In this study what was the probability of dying younger than 70 years old? Did you respond with an experimental probability or a theoretical one?

Answers

1. $\frac{1}{6}$; theoretical   2. $\frac{1}{4}$ 3. $\frac{26}{40}$; $\frac{3}{6}$ 4. $\frac{1}{3}$; theoretical

5. experimental   6. $\frac{39+241}{310,000,000}$ 7. $\frac{10+2+18}{107} \approx 0.28$ 8. $\frac{55+52}{300} \approx 35.7\%$

                                      $\approx 0.000\,000\,903$ experimental   experimental

Measures of central tendency are numbers that locate or approximate the "center" of a set of data—that is, a "typical" value that describes the set of data. Mean and median are the most common measures of central tendency.

The mean is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set. The median is the middle number in a set of data arranged numerically. An outlier is a number that is much smaller or larger than most of the others in the data set. The range of a data set is the difference between the highest and lowest values of the data set.

For additional information, see the Math Notes boxes in Lessons 1.1.3 and 1.1.4 of the *Core Connections, Course 2* text.

The mean is calculated by finding the sum of the data set and dividing it by the number of elements in the set.

Example 1

Find the mean of this set of data: 34, 31, 37, 44, 38, 34, 42, 34, 43, and 41.

- $34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 41 = 378$

- $\frac{378}{10} = 37.8$

The mean of this set of data is 37.8.

Example 2

Find the mean of this set of data: 92, 82, 80, 92, 78, 75, 95, and 77.

- $92 + 82 + 80 + 92 + 78 + 75 + 95 + 77 = 671$

- $\frac{671}{8} = 83.875$

The mean of this set of data is 83.875.

Problems

Find the mean of each set of data.

1. 29, 28, 34, 30, 33, 26, and 34.

2. 25, 34, 35, 27, 31, and 30.

3. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.

4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The median is the middle number in a set of data *arranged in numerical order*. If there is an even number of values, the median is the mean (average) of the two middle numbers.

Example 3

Find the median of this set of data: 34, 31, 37, 44, 38, 34, 43, and 41.

- Arrange the data in order: 31, 34, 34, 37, 38, 41, 43, 44.

- Find the middle value(s): 37 and 38.

- Since there are two middle values, find their mean: $37 + 38 = 75$, $\frac{75}{2} = 37.5$. Therefore, the median of this data set is 37.5.

Example 4

Find the median of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

- Arrange the data in order: 75, 77, 77, 78, 80, 82, 92, 92, 95.

- Find the middle value(s): 80. Therefore, the median of this data set is 80.

Problems

Find median of each set of data.

5. 29, 28, 34, 30, 33, 26, and 34.

6. 25, 34, 27, 25, 31, and 30.

7. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.

8. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The range of a set of data is the difference between the highest value and the lowest value.

Example 5

Find the range of this set of data: 114, 109, 131, 96, 140, and 128.

- The highest value is 140.

- The lowest value is 96.

- $140 - 96 = 44$

- The range of this set of data is 44.

Example 6

Find the range of this set of data: 37, 44, 36, 29, 78, 15, 57, 54, 63, 27, and 48.

- The highest value is 78.

- The lowest value is 27.

- $78 - 27 = 51$

- The range of this set of data is 51.

Problems

Find the range of each set of data in problems 5 through 8.

Outliers are numbers in a data set that are either much higher or much lower that the other numbers in the set.

Example 7

Find the outlier of this set of data: 88, 90 96, 93, 87, 12, 85, and 94.

- The outlier is 12.

Example 8

Find the outlier of this set of data: 67, 54, 49, 76, 64, 59, 60, 72, 123, 44, and 66.

- The outlier is 123.

Problems

Identify the outlier in each set of data.

9. 70, 77, 75, 68, 98, 70, 72, and 71.

10. 14, 22, 17, 61, 20, 16, and 15.

11. 1376, 1645, 1783, 1455, 3754, 1790, 1384, 1643, 1492, and 1776.

12. 62, 65, 93, 51, 55, 14, 79, 85, 55, 72, 78, 83, 91, and 76.

Answers

1. 30.57

2. $30.\overline{3}$

3. $86.1\overline{3}$

4. $106.\overline{6}$

5. median: 30; range: 8

6. median: 28.5; range: 9

7. median: 82; range: 47

8. median: 107; range: 27

9. 98

10. 61

11. 3754

12. 14

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The axis (or axes) of a graph must be marked with equal-sized spaces called intervals. Marking the uniform intervals on the axes is called scaling the axes. The difference between consecutive markings tells the size (scale) of each interval. Note that each axis of a two-dimensional graph may use a different scale.

Sometimes the axis or set of axes is not provided. A student must count the number of usable spaces on the graph paper. How many spaces are usable depends in part on how large the graph will be and how much space will be needed for labeling beside each axis.

Follow these steps to scale each axis of a graph.

1. Find the difference between the smallest and largest numbers (the range) you need to put on an axis.

2. Count the number of intervals (spaces) you have on your axis.

3. Divide the range by the number of intervals to find the interval size.

4. Label the marks on the axis using the interval size.

Sometimes dividing the range by the number of intervals produces an interval size that makes it difficult to interpret the location of points on the graph. The student may then exercise judgment and round the interval size up (always up, if rounded at all) to a number that is convenient to use. Interval sizes like 1, 2, 5, 10, 20, 25, 50, 100, etc., work well. For more information, see the Math Notes box in Lesson 1.2.2 of *the Core Connections, Course 2* text.

Example 1

1. The difference between 0 and 60 is 60.

2. The number line is divided into 5 equal intervals.

3. 60 divided by 5 is 12.

4. The marks are labeled with multiples of the interval size 12.
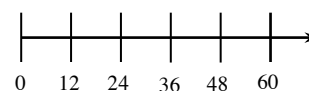
Example 2

1. The difference between 300 and 0 is 300.

2. There are 4 intervals.

3. $300 \div 4 = 75$

4. The axis is labeled with multiples of 75.
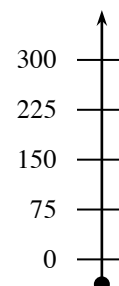
Example 3

1. The difference on the vertical axis is $750 - 0 = 750$. (The origin is $(0, 0)$.) On the horizontal axis the range is $6 - 0 = 6$.

2. There are 5 spaces vertically and 3 spaces horizontally.

3. The vertical interval size is $750 \div 5 = 150$. The horizontal interval is $6 \div 3 = 2$.

4. The axes are labeled appropriately.
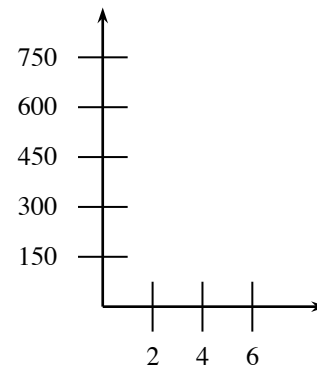
Example 4
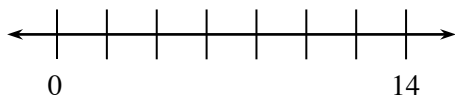
Sometimes the axes extend in the negative direction.

1. The range is $20 - (-15) = 35$.

2. There are 7 intervals along the line.

3. $35 \div 7 = 5$

4. Label the axes with multiples of five.
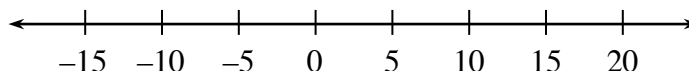
Problems

Scale each axis:

1.

2.

3.

4.

5.

6.

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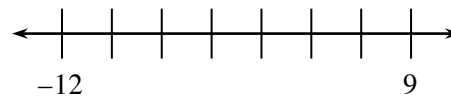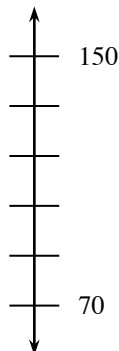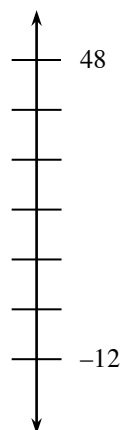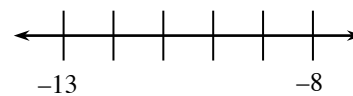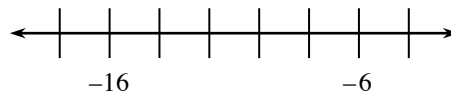
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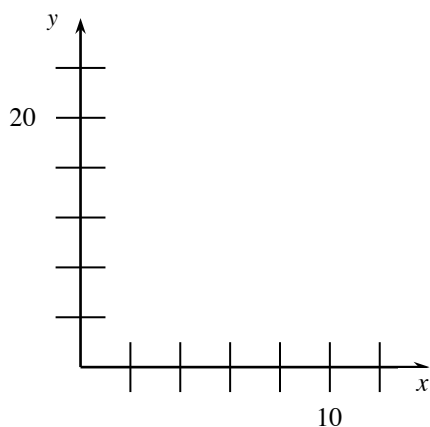
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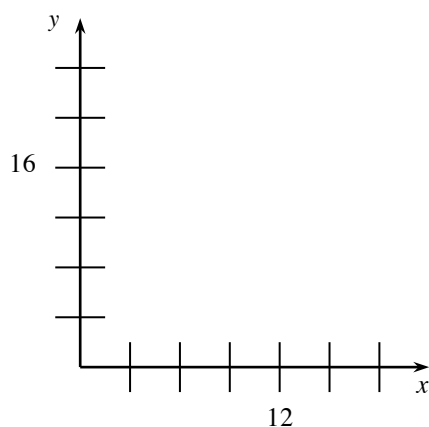
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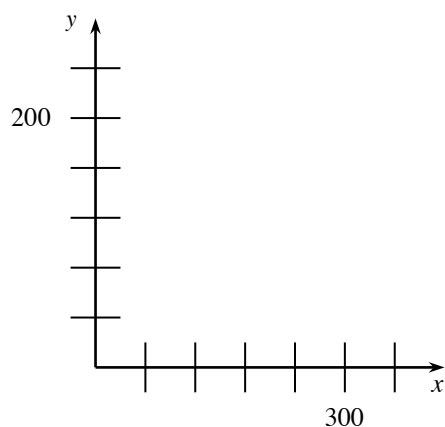
10. Use fractions.
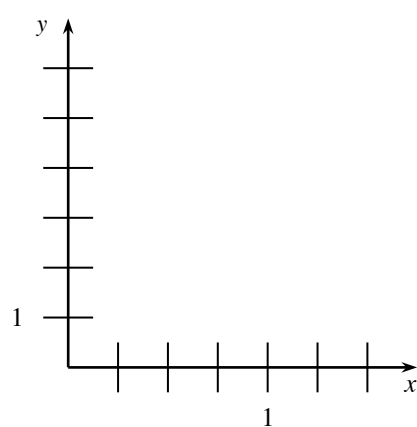
Answers

1. 2, 4, 6, 8, 10, 12

2. –9, –6, –3, 0, 3, 6

3. 86, 102, 118, 134

4. –2, 8, 18, 28, 38

5. –12, –11, –10, –9

6. –18, –14, –12, –10, –8

7. x: 2, 4, 6, 8, 12
    y: 4, 8, 12, 16, 24

8. x: 3, 6, 9, 15, 18
    y: 4, 8, 12, 20, 24

9. x: 60, 120, 180, 240, 360
    y: 40, 80, 120, 160, 240

10. x: $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $1\frac{1}{4}$, $1\frac{1}{2}$
    y: $\frac{1}{2}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3

Fractions that name the same value are called equivalent fractions, such as $\frac{2}{3} = \frac{6}{9}$. One method for finding equivalent fractions is to use the Multiplicative Identity (Identity Property of Multiplication), that is, multiplying the given fraction by a form of the number 1 such as $\frac{2}{2}$, $\frac{5}{5}$, etc. In this course we call these fractions a "Giant One." Multiplying by 1 does not change the value of a number.

For additional information, see the Math Notes box in Lesson 1.2.8 of the *Core Connections, Course 2* text.

Example 1

Find three equivalent fractions for $\frac{1}{2}$.

$$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4} \qquad\qquad \frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6} \qquad\qquad \frac{1}{2} \cdot \frac{4}{4} = \frac{4}{8}$$

Example 2

Use the Giant One to find an equivalent fraction to $\frac{7}{12}$ using 96ths: $\frac{7}{12} \cdot \boxed{1} = \frac{?}{96}$

Which Giant One do you use?

Since $\frac{96}{12} = 8$, the Giant One is $\frac{8}{8}$: $\qquad \frac{7}{12} \cdot \frac{8}{8} = \frac{56}{96}$

Problems

Use the Giant One to find the specified equivalent fraction. Your answer should include the Giant One you use and the equivalent numerator.

1. $\frac{4}{3} \cdot \boxed{1} = \frac{?}{15}$

2. $\frac{5}{9} \cdot \boxed{1} = \frac{?}{36}$

3. $\frac{9}{2} \cdot \boxed{1} = \frac{?}{38}$

4. $\frac{3}{7} \cdot \boxed{1} = \frac{?}{28}$

5. $\frac{5}{3} \cdot \boxed{1} = \frac{?}{18}$

6. $\frac{6}{5} \cdot \boxed{1} = \frac{?}{15}$

Answers

1. $\frac{5}{5}$, 20   2. $\frac{4}{4}$, 20   3. $\frac{19}{19}$, 171   4. $\frac{4}{4}$, 12   5. $\frac{6}{6}$, 30   6. $\frac{3}{3}$, 18

Core Connections, Course 2

ADDITION AND SUBTRACTION OF FRACTIONS

Before fractions can be added or subtracted, the fractions must have the same denominator, that is, a common denominator. We will present two methods for adding or subtracting fractions.

AREA MODEL METHOD

Step 1: Copy the problem.

$\frac{1}{3} + \frac{1}{2}$

Step 2: Draw and divide equal-sized rectangles for each fraction. One rectangle is cut vertically into an equal number of pieces based on the first denominator (bottom). The other is cut horizontally, using the second denominator. The number of shaded pieces in each rectangle is based on the numerator (top). Label each rectangle, with the fraction it represents.

$\frac{1}{3} \quad + \quad \frac{1}{2}$
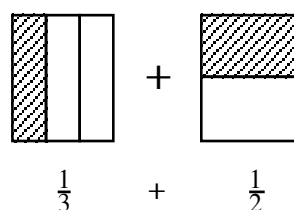
Step 3: Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one.

$\frac{2}{6} \quad + \quad \frac{3}{6}$
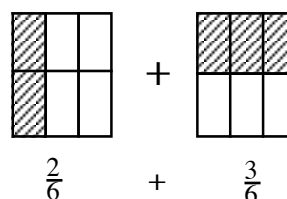
Step 4: Rename the fractions as sixths, because the new rectangles are divided into six equal parts. Change the numerators to match the number of sixths in each figure.
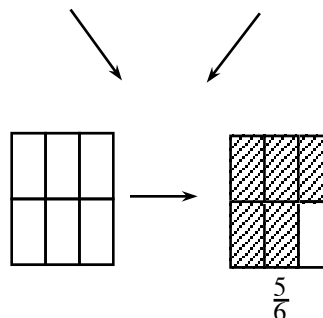
Step 5: Draw an empty rectangle with sixths, then combine all sixths by shading the same number of sixths in the new rectangle as the total that were shaded in both rectangles from the previous step.
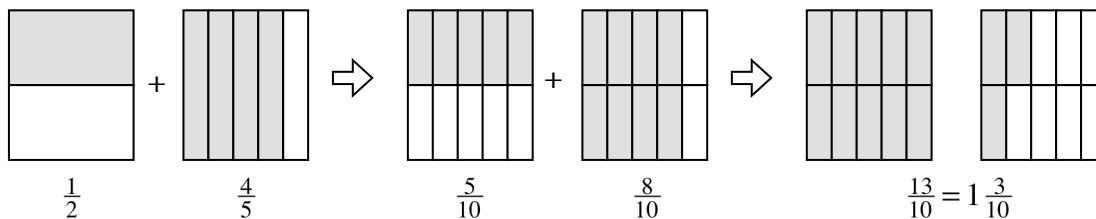
$\frac{5}{6}$

Example 1

$\frac{1}{2} + \frac{2}{5}$ can be modeled as:

$$\frac{1}{2} \qquad \frac{2}{5}$$

 + $\frac{5}{10} + \frac{4}{10}$ so   ⇨
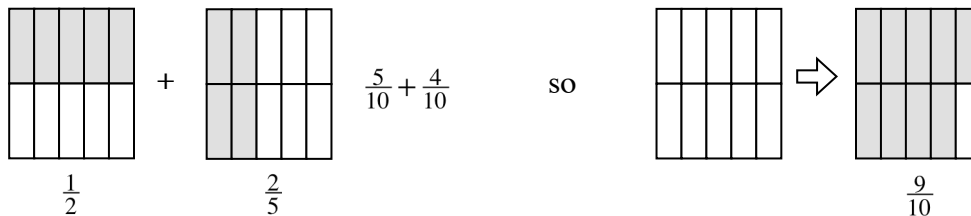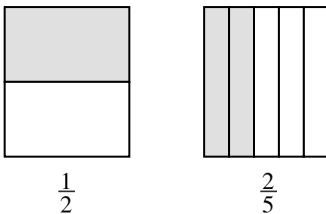
$$\frac{1}{2} \qquad \frac{2}{5} \qquad\qquad\qquad \frac{9}{10}$$

Thus, $\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$.

Example 2

$\frac{1}{2} + \frac{4}{5}$ would be:

 + ⇨ + ⇨

$$\frac{1}{2} \qquad \frac{4}{5} \qquad\qquad \frac{5}{10} \qquad \frac{8}{10} \qquad\qquad \frac{13}{10} = 1\frac{3}{10}$$

Problems

Use the area model method to add the following fractions.

1. $\frac{3}{4} + \frac{1}{5}$

2. $\frac{1}{3} + \frac{2}{7}$

3. $\frac{2}{3} + \frac{3}{4}$

Answers

1. $\frac{19}{20}$

2. $\frac{13}{21}$

3. $\frac{17}{12} = 1\frac{5}{12}$

  Core Connections, Course 2
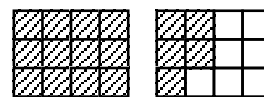
In every Diamond Problem, the product of the two side numbers (left and right) is the top number and their sum is the bottom number.

Diamond Problems are an excellent way of practicing addition, subtraction, multiplication, and division of positive and negative integers, decimals and fractions. They have the added benefit of preparing students for factoring binomials in algebra.
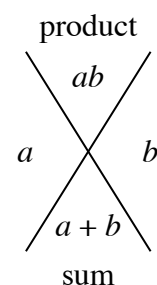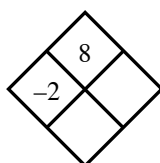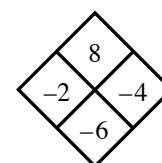
product
ab
a b
$a + b$
sum
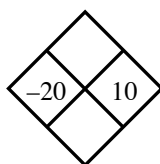
Example 1

The top number is the product of –20 and 10, or –200. The bottom number is the sum of –20 and 10, or –20 + 10 = –10.
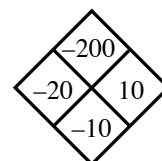
Example 2

The product of the right number and –2 is 8. Thus, if you divide 8 by –2 you get –4, the right number.
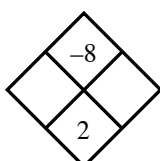The sum of –2 and – 4 is – 6, the bottom number.

Example 3
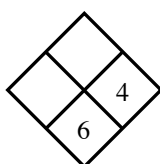
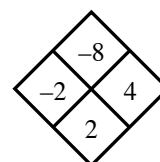To get the left number, subtract 4 from 6, 6 – 4 = 2. The product of 2 and 4 is 8, the top number.
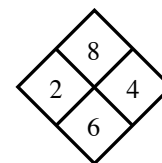
Example 4

The easiest way to find the side numbers in a situation like this one is to look at all the pairs of factors of –8. They are:
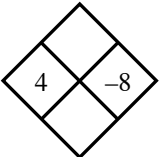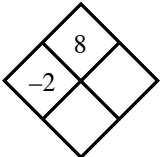
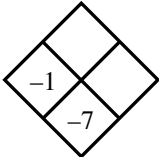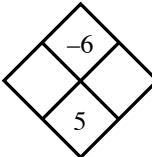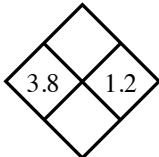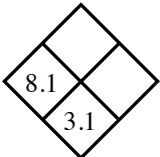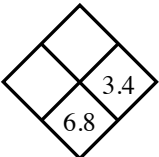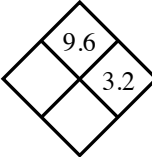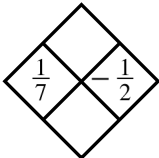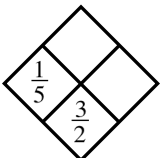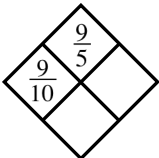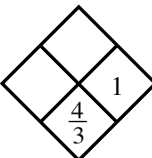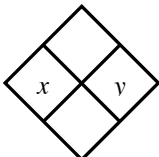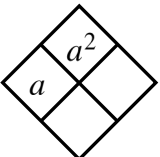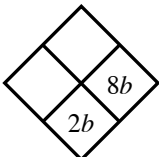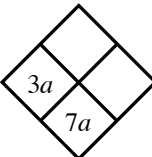–1 and 8, –2 and 4, –4 and 2, and –8 and 1.

Only one of these pairs has a sum of 2: –2 and 4. Thus, the side numbers are –2 and 4.

Problems

Complete each of the following Diamond Problems.

1.

2.

8 / −2

3.

−1 / −7

4.

−6 / 5

5.

3.8 / 1.2

6.

8.1 / 3.1

7.

3.4 / 6.8

8.

9.6 / 3.2

9.

$\frac{1}{7}$ / $-\frac{1}{2}$

10.

$\frac{1}{5}$ / $\frac{3}{2}$

11.

$\frac{9}{5}$ / $\frac{9}{10}$

12.

1 / $\frac{4}{3}$

13.

x / y

14.

a^2 / a

15.

8b / 2b

16.

3a / 7a

Answers

1. −32 and −4

2. −4 and −6

3. −6 and 6

4. 6 and −1

5. 4.56 and 5

6. 5 and 40.5

7. 3.4 and 11.56

8. 3 and 6.2

9. $-\frac{1}{14}$ and $-\frac{5}{14}$

10. $\frac{13}{10}$ and $\frac{13}{50}$

11. $\frac{1}{2}$ and $\frac{7}{5}$

12. $\frac{1}{3}$ and $\frac{1}{3}$

13. xy and $x + y$

14. a and $2a$

15. $-6b$ and $-48b^2$

16. $4a$ and $12a^2$

Core Connections, Course 2

ARITHMETIC OPERATIONS WITH DECIMALS

ADDING AND SUBTRACTING DECIMALS: Write the problem in column form with the decimal points in a vertical column. Write in zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those above.

MULTIPLYING DECIMALS: Multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

DIVIDING DECIMALS: When dividing a decimal by a whole number, place the decimal point in the answer space directly above the decimal point in the number being divided. Divide as with whole numbers. Sometimes it is necessary to add zeros to the number being divided to complete the division.

When dividing decimals or whole numbers by a decimal, the divisor must be multiplied by a power of ten to make it a whole number. The dividend must be multiplied by the same power of ten. Then divide following the same rules for division by a whole number.

For additional information, see the Math Notes boxes in Lessons 3.3.2 and 3.3.3 of the *Core Connections, Course 2* text.

Example 1

Add 47.37, 28.9, 14.56, and 7.8.

$$
\begin{array}{r}
47.37 \\
28.9\mathbf{0} \\
14.56 \\
+\ 7.8\mathbf{0} \\
\hline
98.63
\end{array}
$$

Example 2

Subtract 198.76 from 473.2.

$$
\begin{array}{r}
473.2\mathbf{0} \\
-\ 198.76 \\
\hline
274.44
\end{array}
$$

Example 3

Multiply 27.32 by 14.53.

$$
\begin{array}{r}
27.32 \quad \text{(2 decimal places)} \\
\times\,14.53 \quad \text{(2 decimal places)} \\
\hline
8196 \\
13660 \\
10928 \\
2732 \\
\hline
396.9596 \quad \text{(4 decimal places)}
\end{array}
$$

Example 4

Multiply 0.37 by 0.0004.

$$
\begin{array}{r}
0.37 \quad \text{(2 decimal places)} \\
\times\,0.0004 \quad \text{(4 decimal places)} \\
\hline
\mathbf{0.000}148 \quad \text{(6 decimal places)}
\end{array}
$$

Example 5

Divide 32.4 by 8.

$$
\begin{array}{r}
4.05 \\
8\overline{)\ 32.4\mathbf{0}} \\
\underline{32} \\
0\ 40 \\
\underline{40} \\
0
\end{array}
$$

Example 6

Divide 27.42 by 1.2. First multiply each number by 10^1 or 10.

$$
1.2\overline{)27.42} \Rightarrow 12\overline{)274.2} \Rightarrow 12\overline{)274.20}
$$

$$
\begin{array}{r}
22.85 \\
\underline{24} \\
34 \\
\underline{24} \\
10\ 2 \\
\underline{96} \\
60 \\
\underline{60} \\
0
\end{array}
$$

Problems

1. 4.7 + 7.9

2. 3.93 + 2.82

3. 38.72 + 6.7

4. 58.3 + 72.84

5. 4.73 + 692

6. 428 + 7.392

7. 42.1083 + 14.73

8. 9.87 + 87.47936

9. 9.999 + 0.001

10. 0.0001 + 99.9999

11. 0.0137 + 1.78

12. 2.037 + 0.09387

13. 15.3 + 72.894

14. 47.9 + 68.073

15. 289.307 + 15.938

16. 476.384 + 27.847

17. 15.38 + 27.4 + 9.076

18. 48.32 + 284.3 + 4.638

19. 278.63 + 47.0432 + 21.6

20. 347.68 + 28.00476 + 84.3

21. 8.73 – 4.6

22. 9.38 – 7.5

23. 8.312 – 6.98

24. 7.045 – 3.76

25. 6.304 – 3.68

26. 8.021 – 4.37

27. 14 – 7.431

28. 23 – 15.37

29. 10 – 4.652

30. 18 – 9.043

31. 0.832 – 0.47

32. 0.647 – 0.39

33. 1.34 – 0.0538

34. 2.07 – 0.523

35. 4.2 – 1.764

36. 3.8 – 2.406

37. 38.42 – 32.605

38. 47.13 – 42.703

39. 15.368 + 14.4 – 18.5376

40. 87.43 – 15.687 – 28.0363

41. 7.34 · 6.4

42. 3.71 · 4.03

43. 0.08 · 4.7

44. 0.04 · 3.75

45. 41.6 · 0.302

46. 9.4 · 0.0053

47. 3.07 · 5.4

48. 4.023 · 3.02

49. 0.004 · 0.005

50. 0.007 · 0.0004

51. 0.235 · 0.43

52. 4.32 · 0.0072

53. 0.0006 · 0.00013

54. 0.0005 · 0.00026

55. 8.38 · 0.0001

56. 47.63 · 0.000001

57. 0.078 · 3.1

58. 0.043 · 4.2

59. 350 · 0.004

60. 421 · 0.00005

 Core Connections, Course 2

Divide. Round answers to the hundredth, if necessary.

61. $14.3 \div 8$ 62. $18.32 \div 5$ 63. $147.3 \div 6$

64. $46.36 \div 12$ 65. $100.32 \div 24$ 66. $132.7 \div 28$

67. $47.3 \div 0.002$ 68. $53.6 \div 0.004$ 69. $500 \div 0.004$

70. $420 \div 0.05$ 71. $1.32 \div 0.032$ 72. $3.486 \div 0.012$

73. $46.3 \div 0.011$ 74. $53.7 \div 0.023$ 75. $25.46 \div 5.05$

76. $26.35 \div 2.2$ 77. $6.042 \div 0.006$ 78. $7.035 \div 0.005$

79. $207.3 \div 4.4$ 80. $306.4 \div 3.2$

Answers

|				
1. 12.6	2. 6.75	3. 45.42	4. 131.14	5. 696.73
6. 435.392	7. 56.8383	8. 97.34936	9. 10.000	10. 100.0000
11. 1.7937	12. 2.13087	13. 88.194	14. 115.973	15. 305.245
16. 504.231	17. 51.856	18. 337.258	19. 347.2732	20. 459.98476
21. 4.13	22. 1.88	23. 1.332	24. 3.285	25. 2.624
26. 3.651	27. 6.569	28. 7.63	29. 5.348	30. 8.957
31. 0.362	32. 0.257	33. 1.2862	34. 1.547	35. 2.436
36. 1.394	37. 5.815	38. 4.427	39. 11.2304	40. 43.7067
41. 46.976	42. 14.9513	43. 0.376	44. 0.15	45. 12.5632
46. 0.04982	47. 16.578	48. 12.14946	49. 0.000020	50. 0.0000028
51. 0.10105	52. 0.031104	53. 0.000000078	54. 0.000000130	55. 0.000838
56. 0.0004763	57. 0.2418	58. 0.1806	59. 1.4	60. 0.02105
61. 1.7875 or 1.79	62. 3.664 or 3.66	63. 24.55	64. $3.86\overline{3}$ or 3.86	65. 4.18
66. 4.74	67. 23,650	68. 13,400	69. 125,000	70. 8400
71. 41.25	72. 29.05	73. 4209.09	74. 2334.78	75. 5.04
76. 11.98	77. 1007	78. 1407	79. 47.11	80. 95.75

Fractions, decimals, and percents are different ways to represent the same portion or number.

fraction

words or pictures

decimal

percent

Representations of a Portion
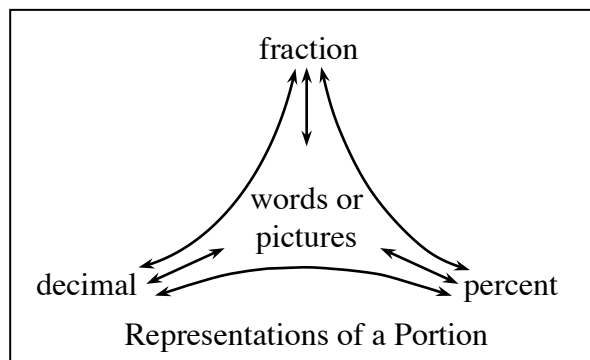
For additional information, see the Math Notes box in Lesson 2.1.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 2 materials.

Examples

Decimal to percent:

Multiply the decimal by 100.

$(0.81)(100) = 81\%$

Fraction to percent:

Write a proportion to find an equivalent fraction using 100 as the denominator. The numerator is the percent.

$$\frac{4}{5} = \frac{x}{100} \quad \text{so} \quad \frac{4}{5} = \frac{80}{100} = 80\%$$

Decimal to fraction:

Use the digits in the decimal as the numerator.
Use the decimal place value name as the denominator. Simplify as needed.

a. $0.2 = \frac{2}{10} = \frac{1}{5}$ b. $0.17 = \frac{17}{100}$

Percent to decimal:

Divide the percent by 100.

$43\% \div 100 = 0.43$

Percent to fraction:

Use 100 as the denominator. Use the percent as the numerator. Simplify as needed.

$$22\% = \frac{22}{100} = \frac{11}{50}$$
$$56\% = \frac{56}{100} = \frac{14}{25}$$

Fraction to decimal:

Divide the numerator by the denominator.

$\frac{3}{8} = 3 \div 8 = 0.375 \qquad \frac{5}{8} = 5 \div 8 = 0.625$

$\frac{3}{11} = 3 \div 11 = 0.2727\ldots = 0.\overline{27}$

To see the process for converting repeating decimals to fractions, see problem 2-22 in the *Core Connections, Course 2* text or the Math Notes box referenced above.

Problems

Convert the fraction, decimal, or percent as indicated.

1. Change $\frac{1}{4}$ to a decimal.

2. Change 50% into a fraction in lowest terms.

3. Change 0.75 to a fraction in lowest terms.

4. Change 75% to a decimal.

5. Change 0.38 to a percent.

6. Change $\frac{1}{5}$ to a percent.

7. Change 0.3 to a fraction.

8. Change $\frac{1}{8}$ to a decimal.

9. Change $\frac{1}{3}$ to a decimal.

10. Change 0.08 to a percent.

11. Change 87% to a decimal.

12. Change $\frac{3}{5}$ to a percent.

13. Change 0.4 to a fraction in lowest terms.

14. Change 65% to a fraction in lowest terms.

15. Change $\frac{1}{9}$ to a decimal.

16. Change 125% to a fraction in lowest terms.

17. Change $\frac{8}{5}$ to a decimal.

18. Change 3.25 to a percent.

19. Change $\frac{1}{16}$ to a decimal. Change the decimal to a percent.

20. Change $\frac{1}{7}$ to a decimal.

21. Change 43% to a fraction. Change the fraction to a decimal.

22. Change 0.375 to a percent. Change the percent to a fraction.

23. Change $\frac{7}{8}$ to a decimal. Change the decimal to a percent.

24. Change $0.\overline{12}$ to a fraction.

25. Change $0.\overline{175}$ to a fraction.

Answers

1. 0.25

2. $\frac{1}{2}$

3. $\frac{3}{4}$

4. 0.75

5. 38%

6. 20%

7. $\frac{3}{10}$

8. 0.125

9. $0.\overline{3}$

10. 8%

11. 0.87

12. 60%

13. $\frac{2}{5}$

14. $\frac{13}{20}$

15. $0.\overline{1}$

16. $\frac{5}{4}$ or $1\frac{1}{4}$

17. 1.6

18. 325%

19. 0.0625; 6.25%

20. $0.\overline{142859}$

21. $\frac{43}{100}$; 0.43

22. $37\frac{1}{2}\%$; $\frac{3}{8}$

23. 0.875; 87.5%

24. $\frac{12}{99} = \frac{4}{33}$

25. $\frac{175}{999}$

     Core Connections, Course 2

ADDITION OF INTEGERS

Students review addition of integers using two concrete models: movement along a number line and positive and negative integer tiles.

To add two integers using a number line, start at the first number and then move the appropriate number of spaces to the right or left depending on whether the second number is positive or negative, respectively. Your final location is the sum of the two integers.

To add two integers using integer tiles, a positive number is represented by the appropriate number of (+) tiles and a negative number is represented by the appropriate number of (–) tiles. To add two integers start with a tile representation of the first integer in a diagram and then place into the diagram a tile representative of the second integer. Any equal number of (+) tiles and (–) tiles makes "zero" and can be removed from the diagram. The tiles that remain represent the sum. For additional information, see the Math Notes box in Lesson 2.2.4 of the *Core Connections, Course 2* text.

Example 1

$-4 + 6$

$-4 + 6 = 2$

Example 2

$-2 + (-4)$

$-2 + (-4) = -6$
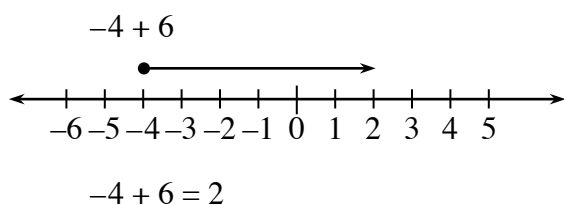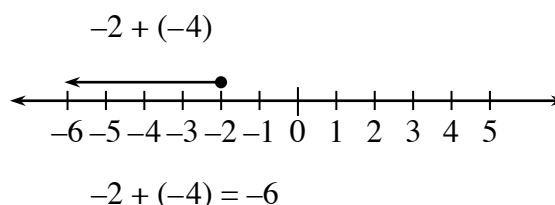
Example 3

$5 + (-6)$

Start with tiles representing the first number.

$+ + + + +$

Add to the diagram tiles representing the second number.

$+ + + + +$
$- \ \ - - \ \ - - \ \ -$

Circle the zero pairs.
–1 is the answer.

$5 + (-6) = -1$

Example 4

$-3 + 7$

$-3 + 7 = 4$

ADDITION OF INTEGERS IN GENERAL

When you add integers using the tile model, zero pairs are only formed if the two numbers have different signs. After you circle the zero pairs, you count the uncircled tiles to find the sum. If the signs are the same, no zero pairs are formed, and you find the sum of the tiles. Integers can be added without building models by using the rules below.

- If the signs are the same, add the numbers and keep the same sign.

- If the signs are different, ignore the signs (that is, use the absolute value of each number.) Subtract the number closest to zero from the number farthest from zero. The sign of the answer is the same as the number that is farthest from zero, that is, the number with the greater absolute value.

Example

For $-4 + 2$, -4 is farther from zero on the number line than 2, so subtract: $4 - 2 = 2$. The answer is -2, since the "4," that is, the number farthest from zero, is negative in the original problem.

Problems

Use either model or the rules above to find these sums.

1. $4 + (-2)$	2. $6 + (-1)$	3. $7 + (-7)$
4. $-10 + 6$	5. $-8 + 2$	6. $-12 + 7$
7. $-5 + (-8)$	8. $-10 + (-2)$	9. $-11 + (-16)$
10. $-8 + 10$	11. $-7 + 15$	12. $-26 + 12$
13. $-3 + 4 + 6$	14. $56 + 17$	15. $7 + (-10) + (-3)$
16. $-95 + 26$	17. $35 + (-6) + 8$	18. $-113 + 274$
19. $105 + (-65) + 20$	20. $-6 + 2 + (-4) + 3 + 5$	21. $5 + (-3) + (-2) + (-8)$
22. $-6 + (-3) + (-2) + 9$	23. $-6 + (-3) + 9$	24. $20 + (-70)$
25. $12 + (-7) + (-8) + 4 + (-3)$	26. $-26 + (-13)$	27. $-16 + (-8) + 9$
28. $12 + (-13) + 18 + (-16)$	29. $50 + (-70) + 30$	30. $19 + (-13) + (-5) + 20$

 Core Connections, Course 2

Answers

1. 2   2. 5   3. 0   4. − 4   5. − 6   6. −5

7. −13   8. −12   9. −27   10. 2   11. 8   12. −14

13. 7   14. 73   15. − 6   16. − 69   17. 37   18. 161

19. 60   20. 0   21. −8   22. −2   23. 0   24. −50

25. −2   26. −39   27. −15   28. 1   29. 10   30. 21

MULTIPLICATION AND DIVISION OF INTEGERS

> Multiply and divide integers two at a time. If the signs are the same, their product will be positive. If the signs are different, their product will be negative.
>
> Follow the same rules for fractions and decimals.
>
> Remember to apply the correct order of operations when you are working with more than one operation.
>
> For additional information, see the Math Notes box in Lesson 3.2.4 of the *Core Connections, Course 2* text.

Examples

a. $2 \cdot 3 = 6$ or $3 \cdot 2 = 6$

b. $-2 \cdot (-3) = 6$ or $(+2) \cdot (+3) = 6$

c. $2 \div 3 = \frac{2}{3}$ or $3 \div 2 = \frac{3}{2}$

d. $(-2) \div (-3) = \frac{2}{3}$ or $(-3) \div (-2) = \frac{3}{2}$

e. $(-2) \cdot 3 = -6$ or $3 \cdot (-2) = -6$

f. $(-2) \div 3 = -\frac{2}{3}$ or $3 \div (-2) = -\frac{3}{2}$

g. $9 \cdot (-7) = -63$ or $-7 \cdot 9 = -63$

h. $-63 \div 9 = -7$ or $9 \div (-63) = -\frac{1}{7}$

             Core Connections, Course 2

Problems

Use the rules above to find each product or quotient.

1. $(-4)(2)$
2. $(-3)(4)$
3. $(-12)(5)$
4. $(-21)(8)$

5. $(4)(-9)$
6. $(13)(-8)$
7. $(45)(-3)$
8. $(105)(-7)$

9. $(-7)(-6)$
10. $(-7)(-9)$
11. $(-22)(-8)$
12. $(-127)(-4)$

13. $(-8)(-4)(2)$
14. $(-3)(-3)(-3)$
15. $(-5)(-2)(8)(4)$
16. $(-5)(-4)(-6)(-3)$

17. $(-2)(-5)(4)(8)$
18. $(-2)(-5)(-4)(-8)$
19. $(-2)(-5)(4)(-8)$
20. $2(-5)(4)(-8)$

21. $10 \div (-5)$
22. $18 \div (-3)$
23. $96 \div (-3)$
24. $282 \div (-6)$

25. $-18 \div 6$
26. $-48 \div 4$
27. $-121 \div 11$
28. $-85 \div 85$

29. $-76 \div (-4)$
30. $-175 \div (-25)$
31. $-108 \div (-12)$
32. $-161 \div 23$

33. $-223 \div (-223)$
34. $354 \div (-6)$
35. $-1992 \div (-24)$
36. $-1819 \div (-17)$

37. $-1624 \div 29$
38. $1007 \div (-53)$
39. $994 \div (-14)$
40. $-2241 \div 27$

Answers

1. -8
2. -12
3. -60
4. -168
5. -36

6. -104
7. -135
8. -735
9. 42
10. 63

11. 176
12. 508
13. 64
14. -27
15. 320

16. 360
17. 320
18. 320
19. -320
20. 320

21. -2
22. -6
23. -32
24. -47
25. -3

26. -12
27. -11
28. -1
29. 19
30. 7

31. 9
32. 7
33. 1
34. -59
35. 83

36. 107
37. -56
38. -19
39. -71
40. -83

MULTIPLICATION OF FRACTIONS

Multiplication of fractions is reviewed using a rectangular area model. Lines that divide the rectangle to represent one fraction are drawn vertically, and the correct number of parts are shaded. Then lines that divide the rectangle to represent the second fraction are drawn horizontally and part of the shaded region is darkened to represent the product of the two fractions.
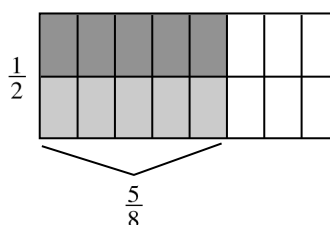
Example 1

$\frac{1}{2} \cdot \frac{5}{8}$ (that is, $\frac{1}{2}$ of $\frac{5}{8}$)

Step 1: Draw a generic rectangle and divide it into 8 pieces vertically. Lightly shade 5 of those pieces. Label it $\frac{5}{8}$.
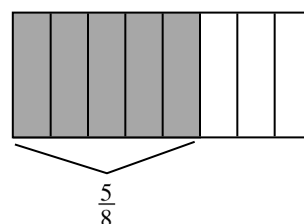
Step 2: Use a horizontal line and divide the generic rectangle in half. Darkly shade $\frac{1}{2}$ of $\frac{5}{8}$ and label it.

Step 3: Write a number sentence.

$$\frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$$

The rule for multiplying fractions derived from the models above is to multiply the numerators, then multiply the denominators. Simplify the product when possible.

For additional information, see the Math Notes box in Lesson 2.2.5 of the *Core Connections, Course 2* text.

 Core Connections, Course 2

Example 2

a. $\frac{2}{3} \cdot \frac{2}{7} \implies \frac{2 \cdot 2}{3 \cdot 7} \implies \frac{4}{21}$

b. $\frac{3}{4} \cdot \frac{6}{7} \implies \frac{3 \cdot 6}{4 \cdot 7} \implies \frac{18}{28} \implies \frac{9}{14}$

Problems

Draw an area model for each of the following multiplication problems and write the answer.

1. $\frac{1}{3} \cdot \frac{1}{6}$

2. $\frac{1}{4} \cdot \frac{3}{5}$

3. $\frac{2}{3} \cdot \frac{5}{9}$

Use the rule for multiplying fractions to find the answer for the following problems. Simplify when possible.

4. $\frac{1}{3} \cdot \frac{2}{5} \cdot$

5. $\frac{2}{3} \cdot \frac{2}{7}$

6. $\frac{3}{4} \cdot \frac{1}{5}$

7. $\frac{2}{5} \cdot \frac{2}{3}$

8. $\frac{2}{3} \cdot \frac{1}{4}$

9. $\frac{5}{6} \cdot \frac{2}{3}$

10. $\frac{4}{5} \cdot \frac{3}{4}$

11. $\frac{2}{15} \cdot \frac{1}{2}$

12. $\frac{3}{7} \cdot \frac{1}{2}$

13. $\frac{3}{8} \cdot \frac{4}{5}$

14. $\frac{2}{9} \cdot \frac{3}{5}$

15. $\frac{3}{10} \cdot \frac{5}{7}$

16. $\frac{5}{11} \cdot \frac{6}{7}$

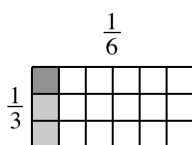17. $\frac{5}{6} \cdot \frac{3}{10}$

18. $\frac{10}{11} \cdot \frac{3}{5}$
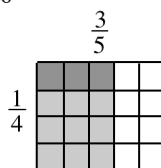
19. $\frac{5}{12} \cdot \frac{3}{5}$

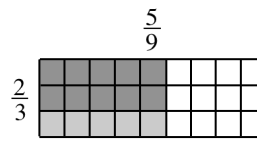20. $\frac{7}{9} \cdot \frac{5}{14}$

Answers

1. $\frac{1}{18}$

2. $\frac{3}{20}$

3. $\frac{10}{27}$

4. $\frac{2}{15}$

5. $\frac{4}{21}$

6. $\frac{3}{20}$

7. $\frac{4}{15}$

8. $\frac{2}{12} = \frac{1}{6}$

9. $\frac{10}{18} = \frac{5}{9}$

10. $\frac{12}{20} = \frac{3}{5}$

11. $\frac{2}{30} = \frac{1}{15}$

12. $\frac{3}{14}$

13. $\frac{12}{40} = \frac{3}{10}$

14. $\frac{6}{45} = \frac{2}{15}$

15. $\frac{15}{70} = \frac{3}{14}$

16. $\frac{30}{77}$

17. $\frac{15}{60} = \frac{1}{4}$

18. $\frac{30}{55} = \frac{6}{11}$

19. $\frac{15}{60} = \frac{1}{4}$

20. $\frac{35}{126} = \frac{5}{18}$

When students are first given expressions like $3 + 4 \cdot 2$, some students think the answer is 14 and some think the answer is 11. This is why mathematicians decided on a method to simplify an expression that uses more than one operation so that everyone can agree on the answer.

There is a set of rules to follow that provides a consistent way for everyone to evaluate expressions. These rules, called the Order of Operations, must be followed in order to arrive at a correct answer. As indicated by the name, these rules state the order in which the mathematical operations are to be completed.

For additional information, see the Math Notes box in Lesson 3.1.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 5 materials.

The first step is to organize the numerical expression into parts called terms, which are single numbers or products of numbers. A numerical expression is made up of a sum or difference of terms.

Examples of numerical terms are: 4, $3(6)$, $6(9 - 4)$, $2 \cdot 3^2$, $3(5 + 2^3)$, and $\frac{16-4}{6}$.

For the problem above, $3 + 4 \cdot 2$, the terms are circled at right. $\boxed{3} + \boxed{4 \cdot 2}$

Each term is simplified separately, giving $3 + 8$. Then the terms are added: $3 + 8 = 11$. Thus, $3 + 4 \cdot 2 = 11$.

Example 1

To evaluate an expression:

$2 \cdot 3^2 + 3(6 - 3) + 10$

- Circle each term in the expression.

$\boxed{2 \cdot 3^2} + \boxed{3(6 - 3)} + \boxed{10}$

- Simplify each term until it is one number by:

  Simplifying the expressions within the parentheses.

  Evaluating each exponential part (e.g., 3^2).

  Multiplying and dividing from left to right.

$\boxed{2 \cdot 3^2} + \boxed{3(3)} + \boxed{10}$

$\boxed{2 \cdot 9} + \boxed{3(3)} + \boxed{10}$

$\boxed{18} + \boxed{9} + \boxed{10}$

- Finally, combine terms by adding or subtracting from left to right.

$27 + 10$

37

 Core Connections, Course 2

Example 2

$$5 - 8 \div 2^2 + 6(5 + 4) - 5^2$$

a. Circle the terms.

a. ⑤ – ⑧÷2² + 6(5 + 4) – 5²

b. Simplify inside the parentheses.

b. ⑤ – ⑧÷2² + 6(9) – 5²

c. Simplify the exponents.

c. ⑤ – ⑧÷4 + 6(9) – 25

d. Multiply and divide from left to right.

d.   5 – 2 + 54 – 25

   Finally, add and subtract from left to right.

          32

Example 3

$$20 + \frac{5+7}{3} - 4^2 + 12 \div 4$$

a. Circle the terms.

a. 20 + 5+7/3 – 4² + 12÷4

b. Multiply and divide left to right, including exponents.

b.   20 + 4 – 16 + 3

   Add or subtract from left to right.

          11

Problems

Circle the terms, then simplify each expression.

1. $5 \cdot 3 + 4$

2. $10 \div 5 + 3$

3. $2(9 - 4) \cdot 7$

4. $6(7 + 3) + 8 \div 2$

5. $15 \div 3 + 7(8 + 1) - 6$

6. $\frac{9}{3} + 5 \cdot 3^2 - 2(14 - 5)$

7. $\frac{20}{6+4} + 7 \cdot 2 \div 2$

8. $\frac{5+30}{7} + 6^2 - 18 \div 9$

9. $2^3 + 8 - 16 \div 8 \cdot 2$

10. $25 - 5^2 + 9 - 3^2$

11. $5(17 - 7) + 4 \cdot 3 - 8$

12. $(5 - 2)^2 + (9 + 1)^2$

13. $4^2 + 9(2) \div 6 + (6 - 1)^2$

14. $\frac{18}{3^2} + \frac{5 \cdot 3}{5}$

15. $3(7 - 2)^2 + 8 \div 4 - 6 \cdot 5$

16. $14 \div 2 + 6 \cdot 8 \div 2 - (9 - 3)^2$

17. $\frac{27}{3} + 18 - 9 \div 3 - (3 + 4)^2$

18. $26 \cdot 2 \div 4 - (6 + 4)^2 + 3(5 - 2)^3$

19. $\left(\frac{42+3}{5}\right)^2 + 3^2 - (5 \cdot 2)^2$

Answers

1. 19

2. 5

3. 70

4. 64

5. 62

6. 30

7. 9

8. 39

9. 12

10. 0

11. 54

12. 109

13. 44

14. 5

15. 47

16. –5

17. –25
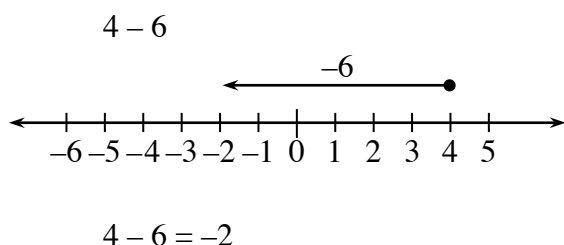
18. – 6

19. –10

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SUBTRACTION OF INTEGERS

Subtraction of integers may also be represented using the concrete models of number lines and (+) and (–) tiles. Subtraction is the opposite of addition so it makes sense to do the opposite actions of addition.
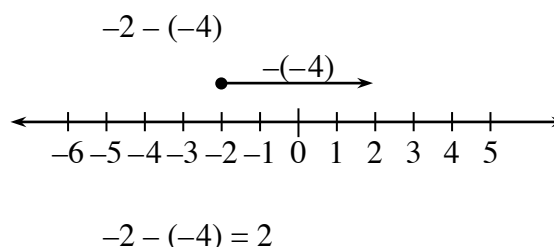
When using the number line, adding a positive integer moves to the right so subtracting a positive integer moves to the left. Adding a negative integer move to the left so subtracting a negative integer moves to the right.

When using the tiles, addition means to place additional tile pieces into the picture and look for zeros to simplify. Subtraction means to remove tile pieces from the picture. Sometimes you will need to place zero pairs in the picture before you have a sufficient number of the desired pieces to remove. For additional information, see the Math Notes box in Lesson 3.2.2 of the *Core Connections, Course 2* text.
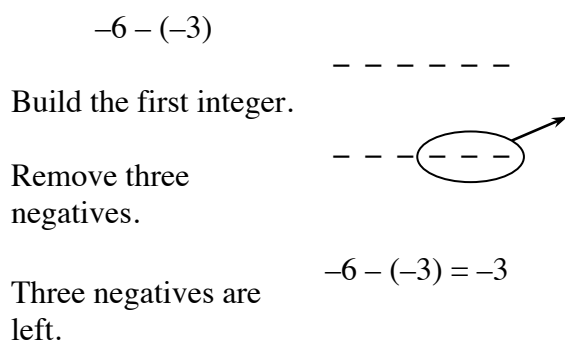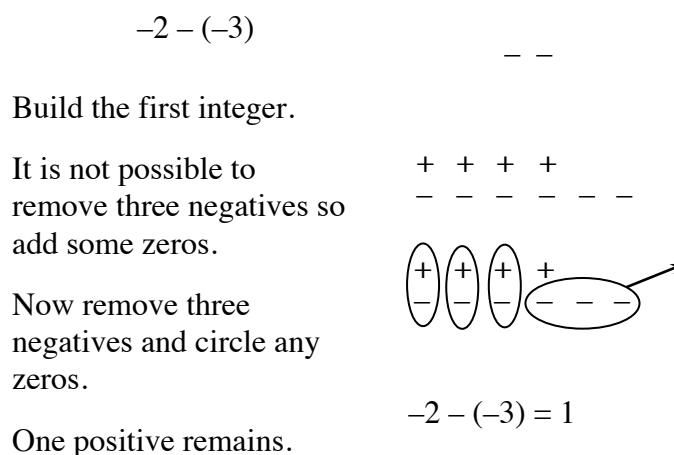
Example 1

$4 - 6$

$4 - 6 = -2$

Example 2

$-2 - (-4)$

$-2 - (-4) = 2$

Example 3

$-6 - (-3)$

Build the first integer.

Remove three negatives.

Three negatives are left.

$-6 - (-3) = -3$

Example 4

$-2 - (-3)$

Build the first integer.

It is not possible to remove three negatives so add some zeros.

Now remove three negatives and circle any zeros.
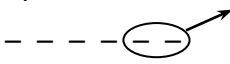
One positive remains.

$-2 - (-3) = 1$
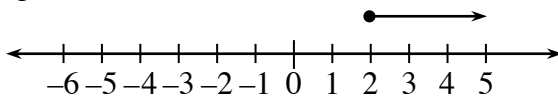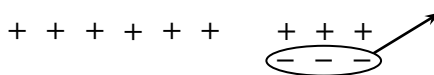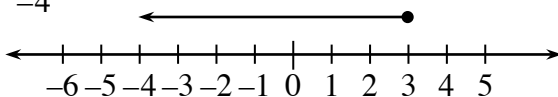
Problems

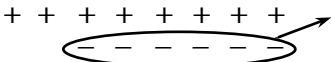Find each difference. Use one of the models for at least the first five differences.

1. $-6 - (-2)$ 2. $2 - (-3)$ 3. $6 - (-3)$

4. $3 - 7$ 5. $7 - (-3)$ 6. $7 - 3$

7. $5 - (3)$ 8. $-12 - (-10)$ 9. $-12 - 10$

10. $12 - (-10)$ 11. $-6 - (-3) - 5$ 12. $6 - (-3) - 5$

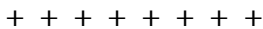13. $8 - (-8)$ 14. $-9 - 9$ 15. $-9 - 9 - (-9)$

Answers (and possible models)

1. -4

2. 5

3. 9

4. -4

5. 10 6. 4 7. 2

8. -2 9. -22 10. 22 11. -8

12. 4 13. 16 14. -18 15. -9

             Core Connections, Course 2

CONNECTING ADDITION AND SUBTRACTION

In the next six examples, compare (a) to (b), (c) to (d), and (e) to (f). Notice that examples (a), (c), and (e) are subtraction problems and examples (b), (d), and (f) are addition problems. The answers to each pair of examples are the same. Also notice that the second integers in the pairs are opposites (that is, they are the same distance from zero on opposite sides of the number line) while the first integers in each pair are the same.

a. $2-(-6)$ $2-(-6)=8$

b. $2+6$ $2+6=8$

c. $-3-(-4)$ $-3-(-4)=1$

d. $-3+4$ $-3+4=1$

e. $-4-(-3)$ $-4-(-3)=-1$

f. $-4+3$ $-4+3=-1$

You can conclude that subtracting an integer is the same as adding its opposite. This fact is summarized below.

SUBTRACTION OF INTEGERS IN GENERAL

To find the difference of two integers, change the subtraction sign to an addition sign. Next change the sign of the integer you are subtracting, and then apply the rules for addition of integers.

For more information on the rules for subtracting integers, see the Math Notes box in Lesson 3.2.3 of the *Core Connections, Course 2* text.

Examples

Use the rule for subtracting integers to find each difference (that is, subtract).

a. $9 - (-12)$ becomes $9 + (+12) = 21$ b. $-9 - (-12)$ becomes $-9 + (+12) = 3$

c. $-9 - 12$ becomes $-9 + (-12) = -21$ d. $9 - 12$ becomes $9 + (-12) = -3$

Problems

Use the rule for subtracting integers to find each difference.

1. $9 - (-3)$ 2. $9 - 3$ 3. $-9 - 3$

4. $-9 - (-3)$ 5. $-14 - 15$ 6. $-16 - (-15)$

7. $-40 - 62$ 8. $-40 - (-62)$ 9. $40 - 62$

10. $40 - (-62)$ 11. $-5 - (-3) - 5 - 6$ 12. $-5 - 3 - (-5) - (-6)$

13. $5 - 3 - (-5) - 6$ 14. $5 - (-4) - 6 - (-7)$ 15. $-125 - (-125) - 125$

16. $5 - (-6)$ 17. $12 - 14$ 18. $20 - 25$

19. $-3 - 2$ 20. $-7 - 3$ 21. $-10 - 5$

22. $-30 - 7$ 23. $-3 - (-3)$ 24. $-3 - (-4)$

25. $10 - (-3)$ 26. $5 - (-9)$ 27. $27 - (-7)$

28. $15 - 32$ 29. $-58 - 37$ 30. $-79 - (-32)$

31. $-62 - 81$ 32. $-106 - 242$ 33. $47 - (-55)$

34. $257 - 349$ 35. $-1010 - (-1010)$

Answers

1.   12   2.   6   3.   −12   4.   −6   5.   −29

6.   −1   7.   −102   8.   22   9.   −22   10. 102

11. −13   12. 3   13. 1   14. 10   15. −125

16. 11   17. −2   18. −5   19. −5   20. −10

21. −15   22. −37   23. 0   24. 1   25. 13

26. 14   27. 34   28. −17   29. −95   30. −47

31. −143   32. −348   33. 102   34. −92   35. 0

                   Core Connections, Course 2

MULTIPLYING DECIMALS AND PERCENTS

> Understanding how many places to move the decimal point in a decimal multiplication problem is connected to the multiplication of fractions and place value.
>
> Computations involving calculating "a percent of a number" are simplified by changing the percent to a decimal.

Example 1

Multiply $(0.2) \cdot (0.3)$.

In fractions this means $\frac{2}{10} \cdot \frac{3}{10} \Rightarrow \frac{6}{100}$.

Knowing that the answer must be in the hundredths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(tenths) = hundredths
Therefore move two places.

$$\begin{array}{r} 0.2 \\ \times\, 0.3 \\ \hline 0.06 \end{array}$$

Example 2

Multiply $(1.7) \cdot (0.03)$.

In fractions this means $\frac{17}{10} \cdot \frac{3}{100} \Rightarrow \frac{51}{1000}$.

Knowing that the answer must be in the thousandths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(hundredths) = thousandths
Therefore move three places.

$$\begin{array}{r} 1.7 \\ \times\, 0.03 \\ \hline 0.051 \end{array}$$

Example 3

Calculate 17% of 32.5 without using a calculator.

Since $17\% = \frac{17}{100} = 0.17$,

17% of $32.5 \Rightarrow (0.17) \cdot (32.5)$

$\Rightarrow 5.525$

$$\begin{array}{r} 32.5 \\ \times\ 0.17 \\ \hline 2275 \\ 3250 \\ \hline 5.525 \end{array}$$

Problems

Identify the number of places to the left to move the decimal point in the product. Do not compute the product.

1. $(0.3) \cdot (0.5)$
2. $(1.5) \cdot (0.12)$
3. $(1.23) \cdot (2.6)$
4. $(0.126) \cdot (3.4)$
5. $17 \cdot (32.016)$
6. $(4.32) \cdot (3.1416)$

Compute without using a calculator.

7. $(0.8) \cdot (0.03)$
8. $(3.2) \cdot (0.3)$
9. $(1.75) \cdot (0.09)$
10. $(4.5) \cdot (3.2)$
11. $(1.8) \cdot (0.032)$
12. $(7.89) \cdot (6.3)$
13. 8% of 540
14. 70% of 478
15. 37% of 4.7
16. 17% of 96
17. 15% of 4.75
18. 130% of 42

Answers

1. 2
2. 3
3. 3
4. 4
5. 3
6. 6
7. 0.024
8. 0.96
9. 0.1575
10. 14.4
11. 0.0576
12. 49.707
13. 43.2
14. 334.6
15. 1.739
16. 16.32
17. 0.7125
18. 54.6

 Core Connections, Course 2

DIVISION BY FRACTIONS

Division by fractions introduces three methods to help students understand how dividing by fractions works. In general, think of division for a problem like $8 \div 2$ as, "In 8, how many groups of 2 are there?" Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, "In $\frac{1}{2}$, how many fourths are there?"

For more information, see the Math Notes box in Lesson 3.3.1 of the *Core Connections, Course 2* text. The first two examples show how to divide fractions using a diagram.

Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.
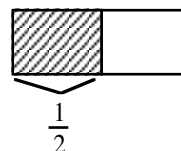
Step 2: Then divide the *original* rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are 2 fourths.
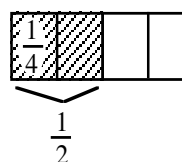
Step 3: Write the equation.

$$\frac{1}{2} \div \frac{1}{4} = 2$$

Example 2

In $\frac{3}{4}$, how many $\frac{1}{2}$s are there?

That is, $\frac{3}{4} \div \frac{1}{2} = ?$

Start with $\frac{3}{4}$.
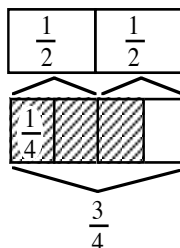
In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one-half).

So: $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$
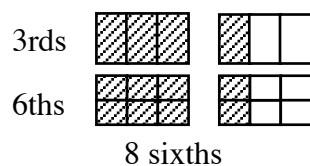(one and one-half halves)
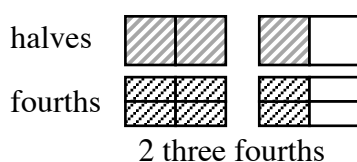
Problems

Use the rectangular model to divide.

1. $1\frac{1}{3} \div \frac{1}{6}$
2. $\frac{3}{2} \div \frac{3}{4}$
3. $1 \div \frac{1}{4}$
4. $1\frac{1}{4} \div \frac{1}{2}$
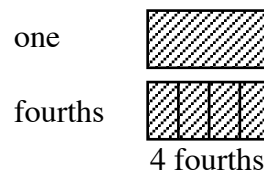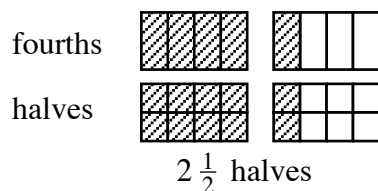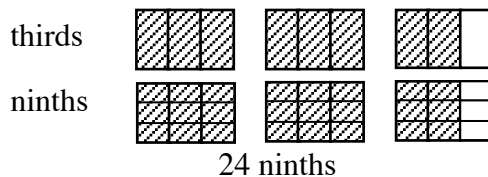5. $2\frac{2}{3} \div \frac{1}{9}$

Answers

1. 8

3rds

6ths

8 sixths

2. 2

halves

fourths

2 three fourths

3. 4

one

fourths

4 fourths

4. $2\frac{1}{2}$

fourths

halves

$2\frac{1}{2}$ halves

5. 24

thirds

ninths

24 ninths

The next two examples use common denominators to divide by a fraction. Express both fractions with a common denominator, then divide the first numerator by the second.

Example 3

$\frac{4}{5} \div \frac{2}{3} =$

$\frac{12}{15} \div \frac{10}{15} =$

$\frac{12}{10} = \frac{6}{5}$ or $1\frac{1}{5}$

Example 4

$1\frac{1}{3} \div \frac{1}{6} =$

$\frac{4}{3} \div \frac{1}{6} =$

$\frac{8}{6} \div \frac{1}{6} = \frac{8}{1}$ or 8

 Core Connections, Course 2

One more way to divide fractions is to use the Giant One from previous work with fractions to create a "Super Giant One." To use a Super Giant One, write the division problem in fraction form, with a fraction in both the numerator and the denominator. Use the reciprocal of the denominator for the numerator and the denominator in the Super Giant One, multiply the fractions as usual, and simplify the resulting fraction when possible.

Example 5

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \boxed{\frac{\frac{4}{1}}{\frac{4}{1}}} = \frac{\frac{4}{2}}{1} = \frac{4}{2} = 2$$

Example 6

$$\frac{\frac{3}{4}}{\frac{1}{6}} \cdot \boxed{\frac{\frac{6}{1}}{\frac{6}{1}}} = \frac{\frac{18}{4}}{1} = \frac{9}{2} = 4\frac{1}{2}$$

Example 7

$$\frac{1\frac{1}{3}}{1\frac{1}{2}} = \frac{\frac{4}{3}}{\frac{3}{2}} \cdot \boxed{\frac{\frac{2}{3}}{\frac{2}{3}}} = \frac{\frac{8}{9}}{1} = \frac{8}{9}$$

Example 8

$$\frac{2}{3} \div \frac{3}{5} \Rightarrow \frac{10}{15} \div \frac{9}{15} \Rightarrow \frac{10}{9}$$

Compared to:

$$\frac{\frac{2}{3}}{\frac{3}{5}} \cdot \boxed{\frac{\frac{5}{3}}{\frac{5}{3}}} = \frac{\frac{10}{9}}{1} = \frac{10}{9} = 1\frac{1}{9}$$

Problems

Complete the division problems below. Use any method.

1. $\frac{3}{7} \div \frac{1}{8}$ 2. $1\frac{3}{7} \div \frac{1}{2}$ 3. $\frac{4}{7} \div \frac{1}{3}$ 4. $1\frac{4}{7} \div \frac{1}{3}$ 5. $\frac{6}{7} \div \frac{5}{8}$

6. $\frac{3}{10} \div \frac{5}{7}$ 7. $2\frac{1}{3} \div \frac{5}{8}$ 8. $7 \div \frac{1}{3}$ 9. $1\frac{1}{3} \div \frac{2}{5}$ 10. $2\frac{2}{3} \div \frac{3}{4}$

11. $3\frac{1}{3} \div \frac{5}{6}$ 12. $1\frac{1}{2} \div \frac{1}{2}$ 13. $\frac{5}{8} \div 1\frac{1}{4}$ 14. $10\frac{1}{3} \div \frac{1}{6}$ 15. $\frac{3}{5} \div 6$

Answers

1. $3\frac{3}{7}$ 2. $2\frac{6}{7}$ 3. $1\frac{5}{7}$ 4. $4\frac{5}{7}$ 5. $1\frac{13}{35}$

6. $\frac{21}{50}$ 7. $3\frac{11}{15}$ 8. 21 9. $3\frac{1}{3}$ 10. $3\frac{5}{9}$

11. 4 12. 3 13. $\frac{1}{2}$ 14. 62 15. $\frac{1}{10}$

In addition and multiplication, the order of the numbers can be reversed: $2 + 5 = 5 + 2$ and $2 \cdot 5 = 5 \cdot 2$. This is called the Commutative Property. In symbols:

The Commutative Property of Addition states: $a + b = b + a$ and

The Commutative Property of Multiplication states: $a \cdot b = b \cdot a$.

When adding three numbers or multiplying three numbers, the grouping can be changed: $(2 + 3) + 5 = 2 + (3 + 5)$ and $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$. This is the Associative Property. In symbols:

The Associative Property of Addition states: $(a + b) + c = a + (b + c)$ and

The Associative Property of Multiplication states: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

The Distributive Property distributes one operation over another. So far in these courses, students have seen multiplication distributed over addition. In symbols:

For all numbers a, b, and c, $a(b + c) = a \cdot b + a \cdot c$.

For example, $2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$.

For additional information see the Math Notes box in Lesson 4.1.1 of the *Core Connections, Course 2* text.

The properties of multiplication and addition allow calculations to be rearranged. Doing this is helpful when doing calculations mentally. Name the property or reason that justifies each step.

Example 1

Calculate mentally: $4 \cdot (17 \cdot 25)$

Step 1	$= 4 \cdot (25 \cdot 17)$	Commutative Property of Multiplication
Step 2	$= (4 \cdot 25) \cdot 17$	Associative Property of Multiplication
Step 3	$= (100) \cdot 17$	mental math
Step 4	$= 1700$	mental math

Example 2

Calculate mentally: $8(56)$

Step 1	$= 8(50 + 6)$	by renaming 56 as $50 + 6$
Step 2	$= 8(50) + 8(6)$	Distributive Property
Step 3	$= 400 + 48$	mental math
Step 4	$= 448$	mental math

Problems

Listed below are possible steps used to mentally calculate a problem. Give the missing reasons that justify the steps.

1. $15(29) = 15(30 + (-1))$ renamed 29 as $30 + (-1)$

    $15(30 - 1) = 15(30) + 15(-1)$ _____a_____

    $150 + (-15) = 150 + (-10 + -5)$ renamed -15 as $-10 + (-5)$

    $150 + (-10) + (-5) = (150 + (-10)) + (-5)$ _____b_____

    $140 + (-5) = 135$ mental math

2. $386 + 177 + 214 = 386 + 214 + 177$ _____a_____

    $386 + 214 + 177 = (386 + 200) + 14 + 177$ _____b_____

    $586 + 14 + 177 = (586 + 14) + 177$ _____c_____

    $600 + 177 = 777$ mental math

3. $49(12) = 12(49)$ _____a_____

    $12(49) = 12(50 - 1)$ renamed 49 as $50 - 1$

    $12(50 - 1) = 12(50) - 12(1)$ _____b_____

    $12(50) - 12 = (6 \cdot 2)(50) - 12$ renamed 12 as $6 \cdot 2$

    $(6 \cdot 2)(50) - 12 = 6(2 \cdot 50) - 12$ _____c_____

    $6(2 \cdot 50) - 12 = 6(100) - 12$ mental math

    $600 - 12 = 588$ mental math

Answers

1.   a.   Distributive   b.   Associative

2.   a.   Commutative   b.   Associative   c.   Associative

3.   a.   Commutative   b.   Distributive   c.   Associative

Geometric figures can be reduced or enlarged. When this change happens, every length of the figure is reduced or enlarged equally (proportionally), and the measures of the corresponding angles stay the same.

The ratio of any two corresponding sides of the original and new figure is called a scale factor. The scale factor may be written as a percent or a fraction. It is common to write new figure measurements over their original figure measurements in a scale ratio, that is, $\frac{\text{NEW}}{\text{ORIGINAL}}$.

For additional information, see the Math Notes box in Lesson 4.1.2 of the *Core Connections, Course 2* text.

Example 1 using a 200% enlargement

Side length ratios:

$\frac{DE}{AB} = \frac{24}{12} = \frac{2}{1}$

$\frac{FD}{CA} = \frac{26}{13} = \frac{2}{1}$

$\frac{FE}{CB} = \frac{10}{5} = \frac{2}{1}$

original triangle   new triangle
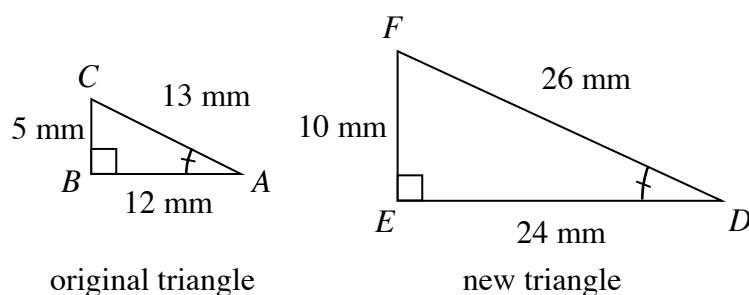
The scale factor for length is 2 to 1.

Example 2

Figures A and B at right are similar. Assuming that Figure A is the original figure, find the scale factor and find the lengths of the missing sides of Figure B.

The scale factor is $\frac{3}{12} = \frac{1}{4}$. The lengths of the missing sides of Figure B are: $\frac{1}{4}(10) = 2.5$, $\frac{1}{4}(18) = 4.5$, and $\frac{1}{4}(20) = 5$.
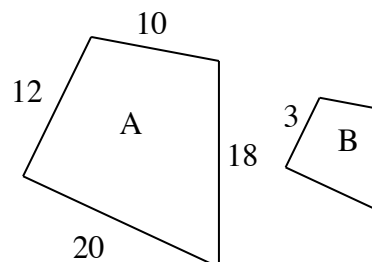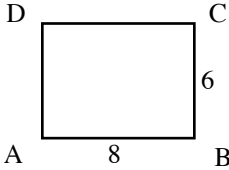
Problems
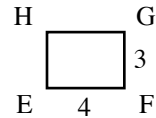
Determine the scale factor for each pair of similar figures in problems 1 through 4.

1.

Original   New

D   C
      6
A   8   B

H   G
    3
E   4   F

2.

Original   New

5
4
8

$1\frac{1}{4}$
1
2

3.

Original   New

3   7
2
6

6   14
4
12

4.

Original   New

9
12

3
4
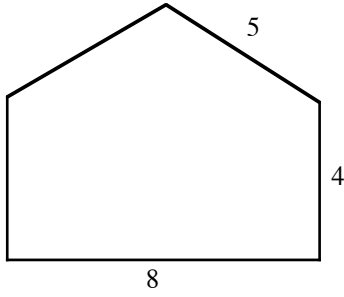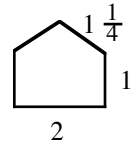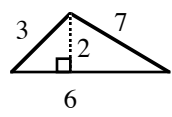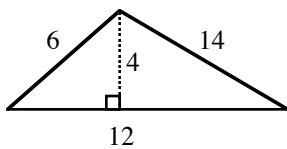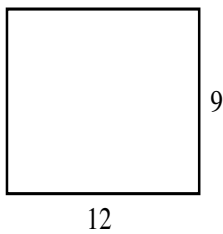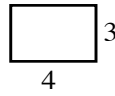
5. A triangle has sides 5, 12, and 13. The triangle was enlarged by a scale factor of 300%.

    a. What are the lengths of the sides of the new triangle?

    b. What is the ratio of the perimeter of the new triangle to the perimeter of the original triangle?

6. A rectangle has a length of 60 cm and a width of 40 cm. The rectangle was reduced by a scale factor of 25%.

    a. What are the dimensions of the new rectangle?

    b. What is the ratio of the perimeter of the new rectangle to the perimeter of the original rectangle?

Answers

1. $\frac{4}{8} = \frac{1}{2}$ 2. $\frac{2}{8} = \frac{1}{4}$

3. $\frac{2}{1}$ 4. $\frac{1}{3}$

5.   a. 15, 36, 39   b. $\frac{3}{1}$ 6.   a. 15 cm and 10 cm   b. $\frac{1}{4}$

A **proportion** is an equation stating the two ratios (fractions) are equal. Two values are in a proportional relationship if a proportion may be set up to relate the values.

For more information, see the Math Notes boxes in Lessons 4.2.3, 4.2.4, and 7.2.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 9 materials.

Example 1

The average cost of a pair of designer jeans has increased $15 in 4 years. What is the unit growth rate (dollars per year)?

Solution: The growth rate is $\frac{15 \text{ dollars}}{4 \text{ years}}$. To create a unit rate we need a denominator of "one."

$\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{x \text{ dollars}}{1 \text{ year}}$. Using a Giant One: $\frac{15 \text{ dollars}}{4 \text{ years}} = \boxed{\frac{4}{4}} \cdot \frac{x \text{ dollars}}{1 \text{ year}} \Rightarrow 3.75 \frac{\text{dollars}}{\text{year}}$.

Example 2

Ryan's famous chili recipe uses 3 tablespoons of chili powder for 5 servings. How many tablespoons are needed for the family reunion needing 40 servings?

Solution: The rate is $\frac{3 \text{ tablespoons}}{5 \text{ servings}}$ so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

$\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3}{5} \cdot \boxed{\frac{8}{8}} = \frac{24}{40} \Rightarrow t = 24$

Another method is to use cross multiplication:

$$\frac{3}{5} = \frac{t}{40}$$

$$\frac{3}{5} \diagdown\!\!\!\!\diagup \frac{t}{40}$$

$$5 \cdot t = 3 \cdot 40$$

$$5t = 120$$

$$t = 24$$

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5} s$ represents the general proportional situation and one could substitute the number of servings needed into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method the answer is 24 tablespoons.

Example 3

Based on the table at right, what is the unit growth rate (meters per year)?

	+2	
height (m)	15	17
years	75	85

+10

Solution: $\dfrac{2\ \text{meters}}{10\ \text{years}} = \dfrac{2\ \text{meters} \cdot \frac{1}{10}}{10\ \text{years} \cdot \frac{1}{10}} = \dfrac{\frac{1}{5}\ \text{meter}}{1\ \text{year}} = \dfrac{1}{5}\ \dfrac{\text{meter}}{\text{year}}$

Problems

For problems 1 through 10 find the unit rate. For problems 11 through 25, solve each problem.

1. Typing 731 words in 17 minutes (words per minute)

2. Reading 258 pages in 86 minutes (pages per minute)

3. Buying 15 boxes of cereal for $43.35 (cost per box)

4. Scoring 98 points in a 40 minute game (points per minute)

5. Buying $2\frac{1}{4}$ pounds of bananas cost $1.89 (cost per pound)

6. Buying $\frac{2}{3}$ pounds of peanuts for $2.25 (cost per pound)

7. Mowing $1\frac{1}{2}$ acres of lawn in $\frac{3}{4}$ of a hour (acres per hour)

8. Paying $3.89 for 1.7 pounds of chicken (cost per pound)

9.
weight (g)	6	8	12	20
length (cm)	15	20	30	50

   What is the weight per cm?

10. For the graph at right, what is the rate in miles per hour?

11. If a box of 100 pencils costs $4.75, what should you expect to pay for 225 pencils?

12. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?

13. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?
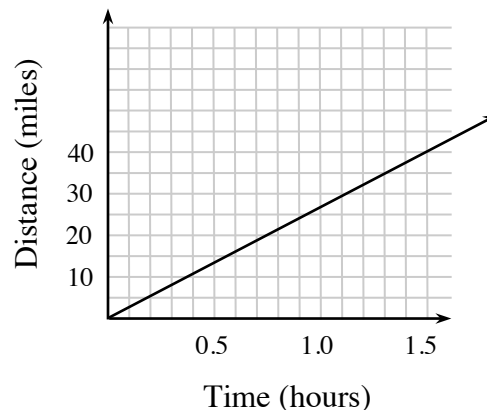
14. The tax on a $600 vase is $54. What should be the tax on a $1700 vase?

15. Use the table at right to determine how long it will take the Spirit club to wax 60 cars.

cars waxed	8	16	32
hours	3	6	12

16. While baking, Evan discovered a recipe that required $\frac{1}{2}$ cups of walnuts for every $2\frac{1}{4}$ cups of flour. How many cups of walnuts will he need for 4 cups of flour?

17. Based on the graph, what would the cost to refill 50 bottles?
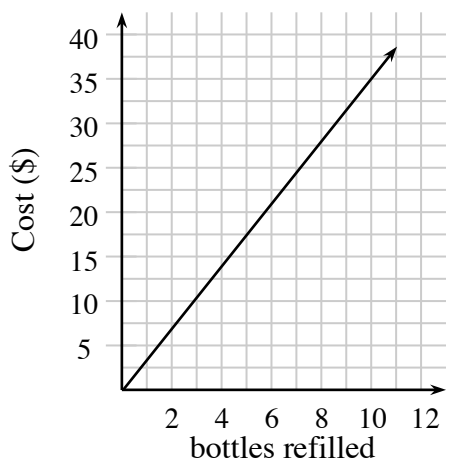
18. Sam grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. How much should he grow in one year?

19. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 60 minutes?

20. If Caitlin needs $1\frac{1}{3}$ cans of paint for each room in her house, how many cans of paint will she need to paint the 7-room house?

21. Stephen receives 20 minutes of video game time every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to work?

22. Sarah's grape vine grew 15 inches in 6 weeks, write an equation to represent its growth after t weeks.

23. On average Max makes 45 out of 60 shots with the basketball, write an equation to represent the average number of shots made out of x attempts.

24. Write an equation to represent the situation in problem 14.

25. Write an equation to represent the situation in problem 17.

Answers

1. $43 \frac{\text{words}}{\text{minute}}$

2. $3 \frac{\text{pages}}{\text{minute}}$

3. $2.89 \frac{\$}{\text{box}}$

4. $2.45 \frac{\text{points}}{\text{minute}}$

5. $0.84 \frac{\$}{\text{pound}}$

6. $3.38 \frac{\$}{\text{pound}}$

7. $2 \frac{\text{acres}}{\text{hour}}$

8. $2.29 \frac{\$}{\text{pound}}$

9. $\frac{2}{5} \frac{\text{grams}}{\text{centimeter}}$

10. $\approx 27 \frac{\text{miles}}{\text{hour}}$

11. $10.69

12. 24.5 min.

13. 19.2 hours

14. $153

15. 22.5 hours

16. $\frac{8}{9}$ cup

17. $175

18. $4\frac{2}{3}$ inches

19. ≈ 5.36 miles

20. $9\frac{1}{3}$ cans

21. $3\frac{3}{8}$ hours

22. $g = \frac{5}{2}t$

23. $s = \frac{3}{4}x$

24. $t = 0.09c$

25. $C = 3.5b$

 Core Connections, Course 2

Rate of change is a ratio that describes how one quantity is changing with respect to another. Unit rate is a rate that compares the change in one quantity to a one-unit change in another quantity. Some examples of rates are miles per hour and price per pound. If 16 ounces of flour cost $0.80 then the unit cost, that is the cost per one ounce, is $\frac{\$0.80}{16} = \$0.05$.

For additional information see the Math Notes box in Lesson 4.2.4 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 9 materials.

Example 1

A rice recipe uses 6 cups of rice for 15 people. At the same rate, how much rice is needed for 40 people?

The rate is: $\frac{6 \text{ cups}}{15 \text{ people}}$ so we need to solve $\frac{6}{15} = \frac{x}{40}$.

The multiplier needed for the Giant One is $\frac{40}{15}$ or $2\frac{2}{3}$.

Using that multiplier yields $\frac{6}{15} \cdot \frac{2\frac{2}{3}}{2\frac{2}{3}} = \frac{16}{40}$ so 16 cups of rice is needed.

Note that the equation $\frac{6}{15} = \frac{x}{40}$ can also be solved using proportions.

Example 2

Arrange these rates from least to greatest:

30 miles in 25 minutes   60 miles in one hour   70 miles in $1\frac{2}{3}$ hour

Changing each rate to a common denominator of 60 minutes yields:

$$\frac{30 \text{ mi}}{25 \text{ min}} = \frac{x}{60} \Rightarrow \frac{30}{25} \cdot \frac{2.4}{2.4} = \frac{72}{60} \frac{\text{mi}}{\text{min}} \quad\quad \frac{60 \text{ mi}}{1 \text{ hr}} = \frac{60 \text{ mi}}{60 \text{ min}} \quad\quad \frac{70 \text{ mi}}{1\frac{2}{3} \text{ hr}} = \frac{70 \text{ mi}}{100 \text{ min}} = \frac{x}{60} \Rightarrow \frac{70}{100} \cdot \frac{0.6}{0.6} = \frac{42 \text{ mi}}{60 \text{ min}}$$

So the order from least to greatest is: 70 miles in $1\frac{2}{3}$ hr < 60 miles in one hour < 30 miles in 25 minutes. Note that by using 60 minutes (one hour) for the common unit to compare speeds, we can express each rate as a unit rate: 42 mph, 60 mph, and 72 mph.

Example 3

A train in France traveled 932 miles in 5 hours. What is the unit rate in miles per hour?

Unit rate means the denominator needs to be 1 hour so: $\frac{932 \text{ mi}}{5 \text{ hr}} = \frac{x}{1 \text{ hr}}$. Solving by using a Giant One of $\frac{0.2}{0.2}$ or simple division yields $x = 186.4$ miles per hour.

Problems

Solve each rate problem below. Explain your method.

1. Balvina knows that 6 cups of rice will make enough Spanish rice to feed 15 people. She needs to know how many cups of rice are needed to feed 135 people.

2. Elaine can plant 6 flowers in 15 minutes. How long will it take her to plant 30 flowers at the same rate?

3. A plane travels 3400 miles in 8 hours. How far would it travel in 6 hours at this rate?

4. Shane rode his bike for 2 hours and traveled 12 miles. At this rate, how long would it take him to travel 22 miles?

5. Selina's car used 15.6 gallons of gas to go 234 miles. At this rate, how many gallons would it take her to go 480 miles?

6. Arrange these readers from fastest to slowest: Abel read 50 pages in 45 minutes, Brian read 90 pages in 75 minutes, and Charlie read 175 pages in 2 hours.

7. Arrange these lunch buyers from greatest to least assuming they buy lunch 5 days per week: Alice spends $3 per day, Betty spends $25 every two weeks, and Cindy spends $75 per month.

8. A train in Japan can travel 813.5 miles in 5 hours. Find the unit rate in miles per hour.

9. An ice skater covered 1500 meters in 106 seconds. Find his unit rate in meters per second.

10. A cellular company offers a price of $19.95 for 200 minutes. Find the unit rate in cost per minute.

11. A car traveled 200 miles on 8 gallons of gas. Find the unit rate of miles per gallon and the unit rate of gallons per mile.

12. Lee's paper clip chain is 32 feet long. He is going to add paper clips continually for the next eight hours. At the end of eight hours the chain is 80 feet long. Find the unit rate of growth in feet per hour.

Answers

1.	54 cups	2.	75 min	3.	2550 miles	4.	$3\frac{2}{3}$ hours
5.	32 gallons	6.	C, B, A	7.	C, A, B	8.	162.7 mi/hour
9.	$\approx$ 14.15 m/s	10.	$\approx$ $0.10/min	11.	25 m/g; $\frac{1}{25}$ g/m	12.	6 ft/hour

 Core Connections, Course 2

Algebraic expressions can be represented by the perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the tile is named by its area as shown on the tile itself in the figures at right. When using the tiles, perimeter is the distance around the exterior of a figure.
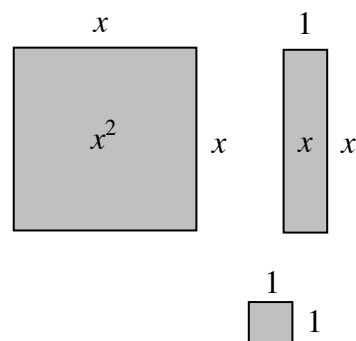
Example 1
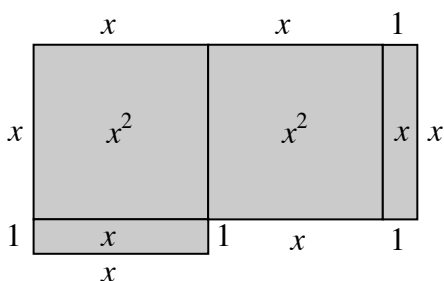
$P = 6x + 4$ units

Example 2
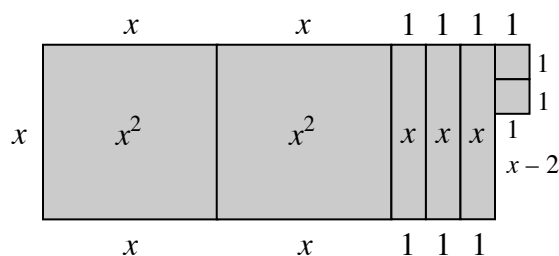
$P = 6x + 8$ units

Problems

Determine the perimeter of each figure.

1.

2.

3.

4.

5.

6.

7.

8.
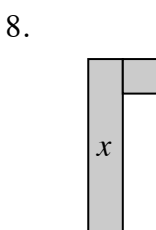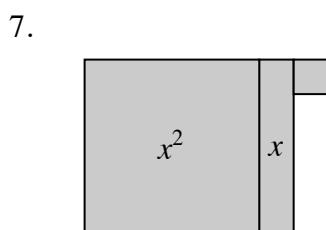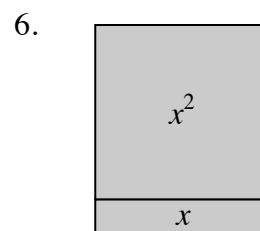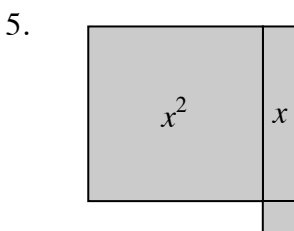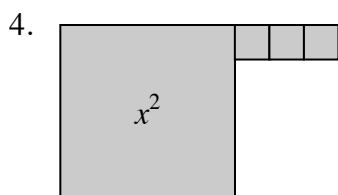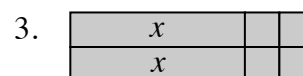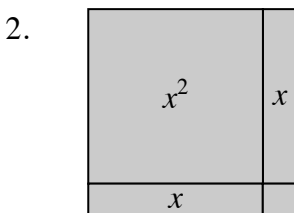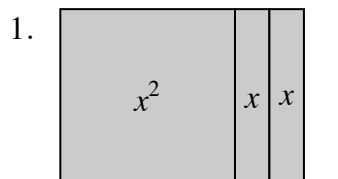
Answers

1. $4x + 4$ un.   2. $4x + 4$ un.   3. $2x + 8$ un.   4. $4x + 6$ un.

5. $4x + 4$ un.   6. $4x + 2$ un.   7. $4x + 4$ un.   8. $2x + 4$ un.

Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) raised to the same powers, into one term. The skill of combining like terms is necessary for solving equations. For additional information, see the Math Notes box in Lesson 4.3.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 7A materials.

Example 1

Combine like terms to simplify the expression $3x + 5x + 7x$.

All these terms have x as the variable, so they are combined into one term, $15x$.

Example 2

Simplify the expression $3x + 12 + 7x + 5$.

The terms with x can be combined. The terms without variables (the constants) can also be combined.

$3x + 12 + 7x + 5$

$3x + 7x + 12 + 5$ Note that in the simplified form the term with the variable is listed before the constant term.

$10x + 17$

Example 3

Simplify the expression $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$.

$5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$

$4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1$

$6x^2 + 8x + 3$

Note that terms with the same variable but with different exponents are not combined and are listed in order of decreasing power of the variable, in simplified form, with the constant term last.

Example 4

The algebra tiles, as shown in the *Algebra Tiles and Perimeter* section, are used to model how to combine like terms.

The large square represents x^2, the rectangle represents x, and the small square represents one. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small squares. If we want to combine: $2x^2 + 3x + 4$ and $3x^2 + 5x + 7$, visualize the tiles to help combine the like terms:

$2x^2$ (2 large squares) + $3x$ (3 rectangles) + 4 (4 small squares)
+ $3x^2$ (3 large squares) + $5x$ (5 rectangles) + 7 (7 small squares)

The combination of the two sets of tiles, written algebraically, is: $5x^2 + 8x + 11$.

Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite *like* terms in vertical columns before you combine them:

$$(2x^2 - 5x + 6) + (3x^2 + 4x - 9)$$

$$\boxed{2x^2} - \boxed{5x} + \boxed{6} + \boxed{3x^2} + \boxed{4x} - \boxed{9}$$

$$
\begin{array}{r}
2x^2 \ \ -5x \ \ +6 \\
+\ 3x^2 \ \ +4x \ \ -9 \\
\hline
5x^2 \ \ -\ x \ \ -3
\end{array}
$$

This procedure may make it easier to identify the terms as well as the sign of each term.

Problems

Combine the following sets of terms.

1. $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$
2. $(3x^2 + x + 4) + (x^2 + 4x + 7)$
3. $(8x^2 + 3) + (4x^2 + 5x + 4)$
4. $(4x^2 + 6x + 5) - (3x^2 + 2x + 4)$
5. $(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$
6. $(3x^2 - 7x) - (x^2 + 3x - 9)$
7. $(5x + 6) + (-5x^2 + 6x - 2)$
8. $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$
9. $3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x$
10. $2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2$

Answers

1. $6x^2 + 8x + 13$
2. $4x^2 + 5x + 11$
3. $12x^2 + 5x + 7$
4. $x^2 + 4x + 1$
5. $6x^2 - 9x - 2$
6. $2x^2 - 10x + 9$
7. $-5x^2 + 11x + 4$
8. $7x + 2$
9. $-c^2 + 4c + x - 3$
10. $3a^3 - 2a^2 + 2a + 14$

 Core Connections, Course 2

The Distributive Property shows how to express sums and products in two ways: $a(b + c) = ab + ac$. This can also be written $(b + c)a = ab + ac$.

Factored form	Distributed form	Simplified form
$a(b + c)$	$a(b) + a(c)$	$ab + ac$

To simplify: Multiply each term on the inside of the parentheses by the term on the outside. Combine terms if possible.

For additional information, see the Math Notes box in Lesson 4.3.3 of the *Core Connections, Course 2* text.

Example 1

$$2(47) = 2(40 + 7)$$
$$= (2 \cdot 40) + (2 \cdot 7)$$
$$= 80 + 14 = 94$$

Example 2

$$3(x + 4) = (3 \cdot x) + (3 \cdot 4)$$
$$= 3x + 12$$

Example 3

$$4(x + 3y + 1) = (4 \cdot x) + (4 \cdot 3y) + 4(1)$$
$$= 4x + 12y + 4$$

Problems

Simplify each expression below by applying the Distributive Property.

1. $6(9 + 4)$

2. $4(9 + 8)$

3. $7(8 + 6)$

4. $5(7 + 4)$

5. $3(27) = 3(20 + 7)$

6. $6(46) = 6(40 + 6)$

7. $8(43)$

8. $6(78)$

9. $3(x + 6)$

10. $5(x + 7)$

11. $8(x - 4)$

12. $6(x - 10)$

13. $(8 + x)4$

14. $(2 + x)5$

15. $-7(x + 1)$

16. $-4(y + 3)$

17. $-3(y - 5)$

18. $-5(b - 4)$

19. $-(x + 6)$

20. $-(x + 7)$

21. $-(x - 4)$

22. $-(-x - 3)$

23. $x(x + 3)$

24. $4x(x + 2)$

25. $-x(5x - 7)$

26. $-x(2x - 6)$

Answers

1. $(6 \cdot 9) + (6 \cdot 4) = 54 + 24 = 78$ 2. $(4 \cdot 9) + (4 \cdot 8) = 36 + 32 = 68$

3. $56 + 42 = 98$ 4. $35 + 20 = 55$ 3. $56 + 42 = 98$ 4. $35 + 20 = 55$

7. $320 + 24 = 344$ 8. $420 + 48 = 468$ 7. $320 + 24 = 344$ 8. $420 + 48 = 468$

11. $8x - 32$ 12. $6x - 60$ 11. $8x - 32$ 12. $6x - 60$

15. $-7x - 7$ 16. $-4y - 12$ 15. $-7x - 7$ 16. $-4y - 12$

19. $-x - 6$ 20. $-x - 7$ 19. $-x - 6$ 20. $-x - 7$

23. $x^2 + 3x$ 24. $4x^2 + 8x$ 23. $x^2 + 3x$ 24. $4x^2 + 8x$

25. $-5x^2 + 7x$ 26. $-2x^2 + 6x$

When the Distributive Property is used to reverse, it is called factoring. Factoring changes a sum of terms (no parentheses) to a product (with parentheses).

$$ab + ac = a(b + c)$$

To factor: Write the common factor of all the terms outside of the parentheses. Place the remaining factors of each of the original terms inside of the parentheses.

Example 4

$4x + 8 = 4 \cdot x + 4 \cdot 2$
$\quad\quad = 4(x + 2)$

Example 5

$6x^2 - 9x = 3x \cdot 2x - 3x \cdot 3$
$\quad\quad = 3x(2x - 3)$

Example 6

$6x + 12y + 3 = 3 \cdot 2x + 3 \cdot 4y + 3 \cdot 1$
$\quad\quad = 3(2x + 4y + 1)$

Problems

Factor each expression below by using the Distributive Property in reverse.

1. $6x + 12$ 2. $5y - 10$ 3. $8x + 20z$ 4. $x^2 + xy$

5. $8m + 24$ 6. $16y + 40$ 7. $8m - 2$ 8. $25y - 10$

9. $2x^2 - 10x$ 10. $21x^2 - 63$ 11. $21x^2 - 63x$ 12. $15y + 35$

13. $4x + 4y + 4z$ 14. $6x + 12y + 6$ 15. $14x^2 - 49x + 28$ 16. $x^2 - x + xy$

 Core Connections, Course 2

Answers

1. $6(x + 2)$
2. $5(y - 2)$
3. $4(2x + 5z)$
4. $x(x + y)$
5. $8(m + 3)$
6. $8(2y + 5)$
7. $2(4m - 1)$
8. $5(5y - 2)$
9. $2x(x - 5)$
10. $21(x^2 - 3)$
11. $21x(x - 3)$
12. $5(3y + 7)$
13. $4(x + y + z)$
14. $6(x + 2y + 1)$
15. $7(2x^2 - 7x + 4)$
16. $x(x - 1 + y)$

Single Region Expression Mats

Algebra tiles and Expression Mats are concrete organizational tools used to represent algebraic expressions. Pairs of Expression Mats can be modified to make Expression Comparison Mats (see next section) and Equation Mats. Positive tiles are shaded and negative tiles are blank. A matching pair of tiles with one tile shaded and the other one blank represents zero (0).

Example 1

Represent $x^2 - 2x + 3$.
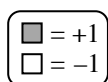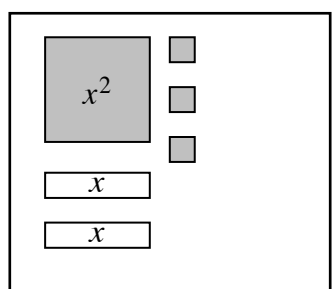
Example 2

Represent $3(x - 2)$.
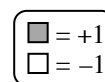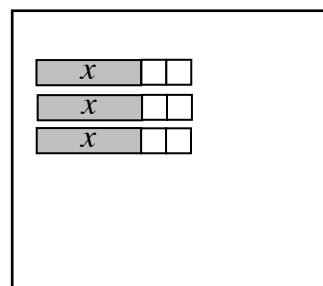
Note that $3(x - 2) = 3x - 6$.

Example 3

This expression makes zero.
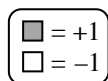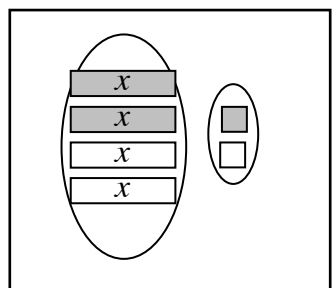
Example 4

Simplify $2x^2 + 2x + 2 + (-2x) + (-3)$.
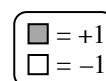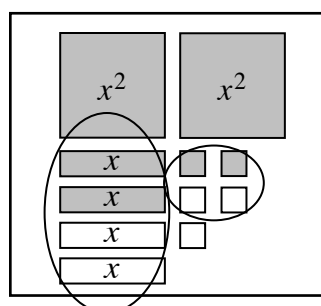
After removing zeros, $2x^2 - 1$ remains.

  Core Connections, Course 2

Problems

$\square = +1$
$\square = -1$

Simplify each expression.
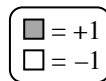
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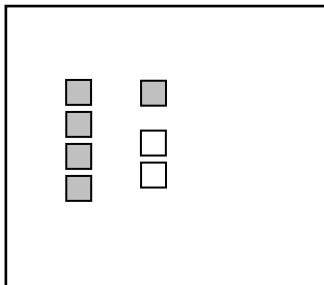
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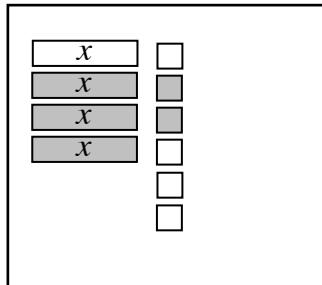
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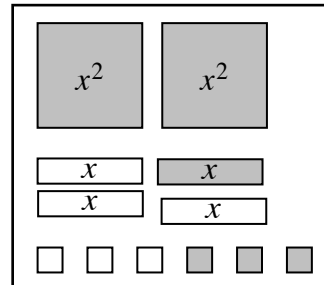
4.
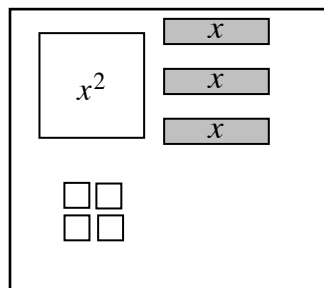
5.
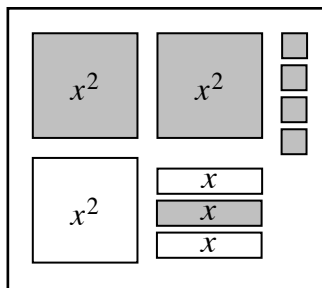
6.
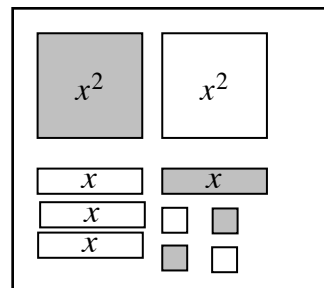
7. $2x - 3 + x + 1$

8. $-3x + 2x + 4$

9. $x^2 - 2x + 3 + 3x - 1$

10. $x + (-3) + 5 - 2x$

11. $-3 + 2x + (-1) - 4x$

12. $3(x + 3) - 2x$

13. $x^2 - 2x + 3 - 2x^2 + 1$

14. $2(x - 2) + 3 - x$

15. $2(x^2 + 3) + 2x - 1$

Answers

1. 3

2. $2x - 2$

3. $2x^2 - 2x$

4. $-x^2 + 3x - 4$

5. $x^2 - x + 4$

6. $-2x$

7. $3x - 2$

8. $-x + 4$

9. $x^2 + x + 2$

10. $-x + 2$

11. $-2x - 4$

12. $x + 9$

13. $-x^2 - 2x + 4$

14. $x - 1$

15. $2x^2 + 2x + 5$

PERCENT PROBLEMS USING DIAGRAMS   5.1.1 and 5.1.2

A variety of percent problems described in words involve the relationship between "the percent," "the part" and "the whole." When this is represented using a number line, solutions may be found using logical reasoning or equivalent fractions (proportions).
These linear models might look like the diagram at right.
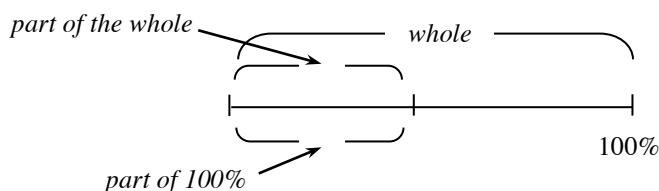
For additional information, see the Math Notes box in Lesson 5.1.2 of the *Core Connections, Course 2* text.

Example 1

Sam's Discount Tires advertises a tire that originally cost $50 on sale for $35. What is the percent discount?

A possible diagram for this situation is shown at right:
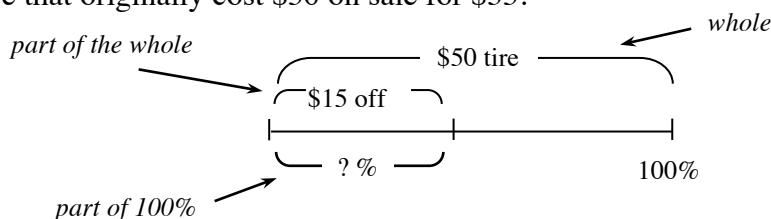
In this situation it is easy to reason that since the percent number total (100%) is twice the cost number total ($50), the percent number saved is twice the cost number saved and is therefore a 30% discount. The problem could also be solved using a proportion $\frac{15}{50} = \frac{?}{100}$.

Example 2

Martin received 808 votes for mayor of Smallville. If this was 32% of the total votes cast, how many people voted for mayor of Smallville?

A possible diagram for this situation is shown at right:
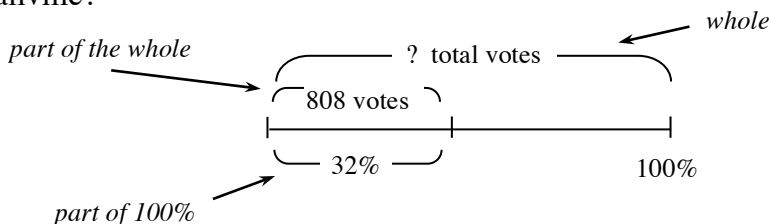
In this case it is better to write a pair of equivalent fractions as a proportion: $\frac{808}{32} = \frac{x}{100}$.

If using the Giant One, the multiplier is $\frac{100}{32} = 3.125$ so $\frac{808}{32} \cdot \frac{3.125}{3.125} = \frac{2525}{100}$.
A total of 2525 people voted for mayor of Smallville.

Note that the proportion in this problem could also be solved using cross-multiplication.

Problems

Use a diagram to solve each of the problems below.

1. Sarah's English test had 90 questions and she got 18 questions wrong. What percent of the questions did she get correct?

2. Cargo pants that regularly sell for $36 are now on sale for 30% off. How much is the discount?

3. The bill for a stay in a hotel was $188 including $15 tax. What percent of the bill was the tax?

4. Alicia got 60 questions correct on her science test. If she received a score of 75%, how many questions were on the test?

5. Basketball shoes are on sale for 22% off. What is the regular price if the sale price is $42?

6. Sergio got 80% on his math test. If he answered 24 questions correctly, how many questions were on the test?

7. A $65 coat is now on sale for $52. What percent discount is given?

8. Ellen bought soccer shorts on sale for $6 off the regular price of $40. What percent did she save?

9. According to school rules, Carol has to convince 60% of her classmates to vote for her in order to be elected class president. There are 32 students in her class. How many students must she convince?

10. A sweater that regularly sold for $52 is now on sale at 30% off. What is the sale price?

11. Jody found an $88 pair of sandals marked 20% off. What is the dollar value of the discount?

12. Ly scored 90% on a test. If he answered 135 questions correctly, how many questions were on the test?

13. By the end of wrestling season, Mighty Max had lost seven pounds and now weighs 128 pounds. What was the percent decrease from his starting weight?

14. George has 245 cards in his baseball card collection. Of these, 85 of the cards are pitchers. What percent of the cards are pitchers?

15. Julio bought soccer shoes at a 35% off sale and saved $42. What was the regular price of the shoes?

Answers

1. 80%	2. $10.80	3. about 8%
4. 80 questions	5. $53.85	6. 30 questions
7. 20%	8. 15%	9. 20 students
10. $36.40	11. $17.60	12. 150 questions
13. about 5%	14. about 35%	15. $120

A ratio is a comparison of two quantities by division. It can be written in several ways:

$$\frac{65 \text{ miles}}{1 \text{ hour}} \text{, 65 miles: 1 hour, or 65 miles to 1 hour}$$

For additional information see the Math Notes box in Lesson 5.1.1 of the *Core Connections, Course 2* text.

Example

A bag contains the following marbles: 7 clear, 8 red, and 5 blue. The following ratios may be stated:

    a.   Ratio of blue to total number of marbles $\Rightarrow \frac{5}{20} = \frac{1}{4}$.

    b.   Ratio of red to clear $\Rightarrow \frac{8}{7}$.

    c.   Ratio of red to blue $\Rightarrow \frac{8}{5}$.

    d.   Ratio of blue to red $\Rightarrow \frac{5}{8}$.

Problems

1.   Molly's favorite juice drink is made by mixing 3 cups of apple juice, 5 cups of cranberry juice, and 2 cups of ginger ale. State the following ratios:

    a.   Ratio of cranberry juice to apple juice.

    b.   Ratio of ginger ale to apple juice.

    c.   Ratio of ginger ale to finished juice drink (the mixture).

2.   A 40-passenger bus is carrying 20 girls, 16 boys, and 2 teachers on a field trip to the state capital. State the following ratios:

    a.   Ratio of girls to boys.   b.   Ratio of boys to girls.

    c.   Ratio of teachers to students.   d.   Ratio of teachers to passengers.

3.   It is important for Molly (from problem one) to keep the ratios the same when she mixes larger or smaller amounts of the drink. Otherwise, the drink does not taste right. If she needs a total of 30 cups of juice drink, how many cups of each liquid should be used?

4.   If Molly (from problem one) needs 25 cups of juice drink, how many cups of each liquid should be used? Remember that the ratios must stay the same.

Answers

1.   a. $\frac{5}{3}$ b. $\frac{2}{3}$ c. $\frac{2}{10} = \frac{1}{5}$ 2.   a. $\frac{20}{16} = \frac{5}{4}$ b. $\frac{16}{20} = \frac{4}{5}$ c. $\frac{2}{36}$ d. $\frac{2}{38}$

3.   9 cups apple juice, 15 cups cranberry juice, 6 cups ginger ale   4. $7\frac{1}{2}$ cups apple, $12\frac{1}{2}$ cups cranberry juice, 5 cups ginger ale

Two events are **independent** if the outcome of one event does not affect the outcome of the other event. For example, if you draw a card from a standard deck of playing cards but replace it before you draw again, the outcomes of the two draws are independent.

Two events are **dependent** if the outcome of one event affects the outcome of the other event. For example, if you draw a card from a standard deck of playing cards and do not replace it for the next draw, the outcomes of the two draws are dependent.

Example 1

Juan pulled a red card from the deck of regular playing cards. This probability is $\frac{26}{52}$ or $\frac{1}{2}$. He puts the card back into the deck. Will his chance of pulling a red card next time change?

No, his chance of pulling a red card next time will not change, because he replaced the card. There are still 26 red cards out of 52. This is an example of an independent event; his pulling out and replacing a red card does not affect any subsequent selections from the deck.

Example 2

Brett has a bag of 30 multi-colored candies. 15 are red, 6 are blue, 5 are green, 2 are yellow, and 2 are brown. If he pulls out a yellow candy and eats it, does this change his probability of pulling any other candy from the bag?

Yes, this changes the probability, because he now has only 29 candies in the bag and only 1 yellow candy. Originally, his probability of yellow was $\frac{2}{30}$ or $\frac{1}{15}$; it is now $\frac{1}{29}$. Similarly, red was $\frac{15}{30}$ or $\frac{1}{2}$ and now is $\frac{15}{29}$, better than $\frac{1}{2}$. This is an example of a dependent event.

Problems

Decide whether these events are independent or dependent events.

1. Flipping a coin, and then flipping it again.

2. Taking a black 7 out of a deck of cards and not returning it, then taking out another card.

3. Taking a red licorice from a bag and eating it, then taking out another piece of licorice.

Answers

1. independent   2. dependent   3. dependent

PROBABILITY OF COMPOUND EVENTS

Sometimes when you are finding a probability, you are interested in either of two outcomes taking place, but not both. For example, you may be interested in drawing a king or a queen from a deck of cards. At other times, you might be interested in one event followed by another event. For example, you might want to roll a one on a number cube and then roll a six. The probabilities of combinations of simple events are called **compound events**.

To find the probability of *either* one event *or* another event that has nothing in common with the first, you can find the probability of each event separately and then add their probabilities. Using the example of drawing a king or a queen from a deck of cards:

$$P(\text{king}) = \tfrac{4}{52} \text{ and } P(\text{queen}) = \tfrac{4}{52} \text{ so } P(\text{king or queen}) = \tfrac{4}{52} + \tfrac{4}{52} = \tfrac{8}{52} = \tfrac{2}{13}$$

For two independent events, to find the probability of *both* one *and* the other event occurring, you can find the probability of each event separately and then multiply their probabilities. Using the example of rolling a one followed by a six on a number cube:

$$P(1) = \tfrac{1}{6} \text{ and } P(6) = \tfrac{1}{6} \text{ so } P(1 \text{ then } 6) = \tfrac{1}{6} \cdot \tfrac{1}{6} = \tfrac{1}{36}$$

Note that you would carry out the same computation if you wanted to know the probability of rolling a one on a green cube, and a six on a red cube, if you rolled both of them at the same time.

Example 1

A spinner is divided into five equal sections numbered 1, 2, 3, 4, and 5. What is the probability of spinning *either* a 2 *or* a 5?

Step 1: Determine both probabilities: $P(2) = \tfrac{1}{5}$ and $P(5) = \tfrac{1}{5}$

Step 2: Since these are *either-or* compound events, add the fractions describing each probability: $\tfrac{1}{5} + \tfrac{1}{5} = \tfrac{2}{5}$

The probability of spinning a 2 or a 5 is $\tfrac{2}{5}$: $P(2 \text{ or } 5) = \tfrac{2}{5}$

Example 2

If each of the regions in each spinner at right is the same size, what is the probability of spinning each spinner and getting a green t-shirt?
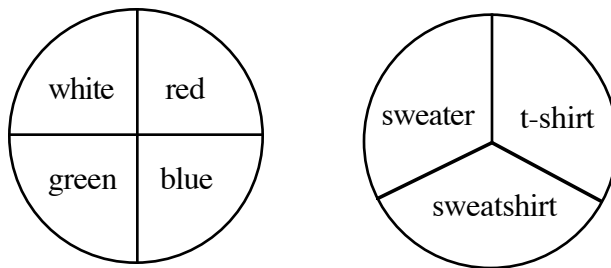
Step 1: Determine both possibilities:
$P(\text{green}) = \frac{1}{4}$ and $P(\text{t-shirt}) = \frac{1}{3}$

Step 2: Since you are interested in the compound event of *both* green *and* a t-shirt, multiply both probabilities: $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

The probability of spinning a green t-shirt is $\frac{1}{12}$: $P(\text{green t-shirt}) = \frac{1}{12}$

Problems

Assume in each of the problems below that events are independent of each other.

1. One die, numbered 1, 2, 3, 4, 5, and 6, is rolled. What is the probability of rolling *either* a 1 *or* a 6?

2. Mary is playing a game in which she rolls one die and spins a spinner. What is the probability she will get *both* the 3 *and* black she needs to win the game?
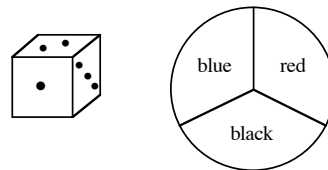
3. A spinner is divided into eight equal sections. The sections are numbered 1, 2, 3, 4, 5, 6, 7, and 8. What is the probability of spinning a 2, 3, or a 4?

4. Patty has a box of 12 colored pencils. There are 2 blue, 1 black, 1 gray, 3 red, 2 green, 1 orange, 1 purple, and 1 yellow in the box. Patty closes her eyes and chooses one pencil. She is hoping to choose a green or a red. What is the probability she will get her wish?

5. Use the spinners at right to tell Paul what his chances are of getting the silver truck he wants.
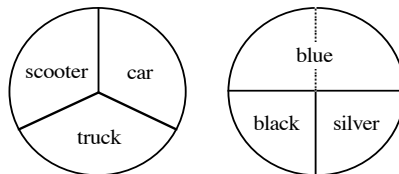
6. On the way to school, the school bus must go through two traffic signals. The first light is green for 25 seconds out of each minute, and the second light is green for 35 seconds out of each minute. What is the probability that both lights will be green on the way to school?

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7. There are 250 students at South Lake Middle School. 125 enjoy swimming, 50 enjoy skateboarding, and 75 enjoy playing softball. Assuming that enjoyment of these activities is independent, what is the probability a student enjoys all three sports?

8. John has a bag of jellybeans. There are 100 beans in the bag. $\frac{1}{4}$ of the beans are cherry, $\frac{1}{4}$ of the beans are orange, $\frac{1}{4}$ of the beans are licorice, and $\frac{1}{4}$ of the beans are lemon. What is the probability that John will chose one of his favorite flavors, orange, or cherry?

9. A nationwide survey showed that only 4% of children liked eating lima beans. What is the probability that any two children will both like lima beans?

Answers

1. $\frac{2}{6}$ or $\frac{1}{3}$

2. $\frac{1}{18}$

3. $\frac{3}{8}$

4. $\frac{5}{12}$

5. $\frac{1}{12}$

6. $\frac{25}{60} \cdot \frac{35}{60} \approx 0.243$

7. $\frac{125}{250} \cdot \frac{50}{250} \cdot \frac{75}{250} = \frac{3}{100}$

8. $\frac{2}{4}$ or $\frac{1}{2}$

9. $\frac{1}{625} = 0.0016$

COUNTING METHODS

There are several different models you can use to determine all possible outcomes for compound events when *both* one event *and* the other occur: a systematic list, a probability table, and a probability tree. See the Math Notes box in Lesson 5.5.2 of the *Core Connections, Course 2* text for details on these three methods.

Not only can you use a **probability table** to help list all the outcomes, but you can also use it to help you determine probabilities of independent compound events when *both* one event *and* the other occur. For example, the following probability table (sometimes called an **area model**) helps determine the probabilities from Example 2 in the previous section:

	$\frac{1}{4}$ white	$\frac{1}{4}$ red	$\frac{1}{4}$ blue	$\frac{1}{4}$ green
$\frac{1}{3}$ sweater				
$\frac{1}{3}$ sweatshirt				
$\frac{1}{3}$ t-shirt				

Each box in the rectangle represents the compound event of *both* a color *and* the type of clothing (sweater, sweatshirt, or t-shirt). The area of each box represents the probability of getting each combination. For example, the shaded region represents the probability of getting a green t-shirt: $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$.

Example 3

At a class picnic Will and Jeff were playing a game where they would shoot a free throw and then flip a coin. Each boy only makes one free throw out of three attempts. Use a probability table (area model) to find the probability that one of the boys makes a free throw, and then flips a head. What is the probability that they miss the free throw and then flip tails?

	H	T
Make		
Miss		
Miss		

By finding the area of the small rectangles, the probabilities are:
$P(\text{make } and \text{ heads}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$, and $P(\text{miss } and \text{ tails}) = \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6}$

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Example 4

Chris owns a coffee cart that he parks outside the downtown courthouse each morning. 65% of his customers are lawyers; the rest are jury members. 60% of Chris's sales include a muffin, 10% include cereal, and the rest are coffee only. What is the probability a lawyer purchases a muffin or cereal?

The probabilities could be represented in an area model as follows:

	lawyer 0.65	jury 0.35
muffin 0.60		
cereal 0.10		
coffee only 0.30		

Probabilities can then be calculated:

	lawyer 0.65	jury 0.35
muffin 0.60	0.39	0.21
cereal 0.10	0.065	0.035
coffee only 0.30	0.195	0.105

The probability a lawyer purchases a muffin or cereal is
0.39 + 0.065 = 0.455 or 45.5%.

Example 5

The local ice cream store has choices of plain, sugar, or waffle cones. Their ice cream choices are vanilla, chocolate, bubble gum, or frozen strawberry yogurt. The following toppings are available for the ice cream cones: sprinkles, chocolate pieces, and chopped nuts.

What are all the possible outcomes for a cone *and* one scoop of ice cream *and* a topping? How many outcomes are possible?

Probability tables are useful only when there are two events. In this situation there are three events (cone, flavor, topping), so we will use a probability tree.

There are four possible flavors, each with three possible cones. Then each of those 12 outcomes can have three possible toppings. There are 36 outcomes for the compound event of choosing a flavor, cone, *and* topping.

Note that the list of outcomes, and the total number of outcomes, does not change if we change the order of events. We could just as easily have chosen the cone first.
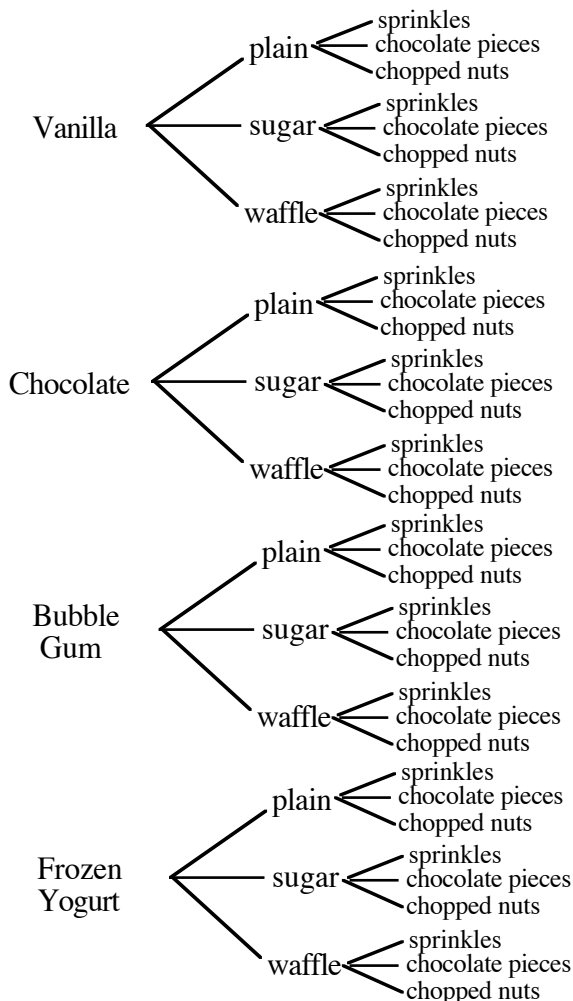
Problems

Use probability tables or tree diagrams to solve these problems.

1. How many different combinations are possible when buying a new bike if the following options are available:

    •   mountain bike or road bike

    •   black, red, yellow, or blue paint

    •   3-speed, 5-speed, or 10-speed

2. A new truck is available with:

    •   standard or automatic transmission

    •   2-wheel or 4-wheel drive

    •   regular or king cab

    •   long or short bed

    How many combinations are possible?

3. A tax assessor categorizes 25% of the homes in how city as having a large backyard, 65% as having a small backyard, and 10% as having no backyard. 30% of the homes have a tile roof, the rest have some other kind of roof. What is the probability a home with a tile roof has a backyard?

4. There is space for only 96 students at University High School to enroll in a shop class: 25 students in woodworking, 25 students in metalworking, and the rest in print shop. Three fourths of the spaces are reserved for seniors, and one fourth are for juniors. What is the probability that a student enrolled in shop class is a senior in print shop? What is the probability that a student enrolled in shop class is a junior in wood or metal shop?

5. Insurance companies use probabilities to determine the rate they will charge for an insurance policy. In a study of 3000 people that had life insurance policies, an insurance company collected the following data of how old people were when they died, compared to how tall they were. In this study, what was the probability of being tall (over 6 ft) and dying under 50 years old? What was the probability of being tall and dying under 70 years old? What was the probability of dying between 50 and 69 years old?

	Age (years)				
	Under 50	50 to 59	60 to 69	70 to 79	80 or over
over 6 ft tall	30	25	52	82	111
under 6 ft tall	270	225	468	738	999

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Answers

1. There are 24 possible combinations as shown below.

Mountain
- black
  - 3 speed
  - 5 speed
  - 10 speed
- red
  - 3 speed
  - 5 speed
  - 10 speed
- yellow
  - 3 speed
  - 5 speed
  - 10 speed
- blue
  - 3 speed
  - 5 speed
  - 10 speed

Road
- black
  - 3 speed
  - 5 speed
  - 10 speed
- red
  - 3 speed
  - 5 speed
  - 10 speed
- yellow
  - 3 speed
  - 5 speed
  - 10 speed
- blue
  - 3 speed
  - 5 speed
  - 10 speed
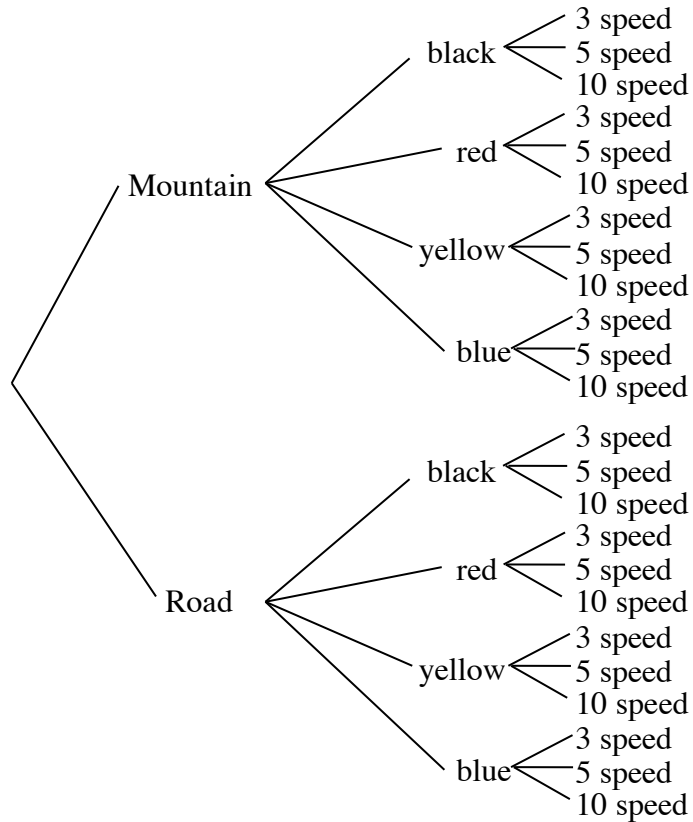
2. There are 16 possible combinations as shown below.

Standard
- 2-wheel drive
  - regular cab
    - long bed
    - short bed
  - king cab
    - long bed
    - short bed
- 4-wheel drive
  - regular cab
    - long bed
    - short bed
  - king cab
    - long bed
    - short bed

Automatic
- 2-wheel drive
  - regular cab
    - long bed
    - short bed
  - king cab
    - long bed
    - short bed
- 4-wheel drive
  - regular cab
    - long bed
    - short bed
  - king cab
    - long bed
    - short bed
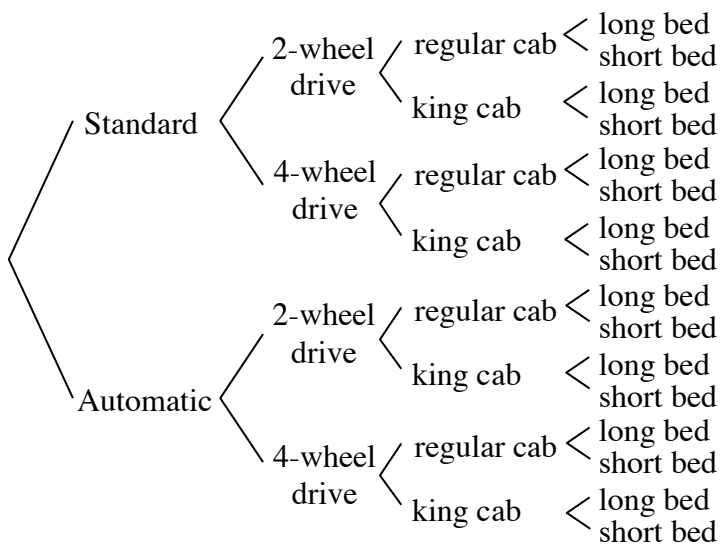
3. The probability is $0.075 + 0.195 = 0.27$ or 27%.

	large yard 25%	small yard 65%	no yard 10%
tile roof 30%	0.075	0.195	
other roof 70%			

4. The probability of a senior in print shop is about 0.359%. The probability of a junior in wood or metal shop is $0.065 + 0.065 \approx 0.13$.

	seniors $\frac{3}{4}$	juniors $\frac{1}{4}$
woodworking $\frac{25}{96}$		≈ 0.065
metalworking $\frac{25}{96}$		≈ 0.065
print shop $\frac{46}{96}$	≈ 0.359	

5. The probability of being tall (over 6 ft) and dying young (under 50 years old) is $\frac{30}{3000} = 0.01$.

   The probability of being tall and dying under 70 years old is $\frac{30+25+52}{3000} \approx 0.036$.

   The probability of dying between 50 and 69 years old is $\frac{25+52+225+468}{3000} \approx 0.257$.

 Core Connections, Course 2

At first students used the 5-D Process to solve problems. However, solving complicated problems with the 5-D Process can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the 5-D Process can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to use the 5-D Process. Most of the problems here will not be complex so that you can practice writing equations using the 5-D Process. The same example problems previously used are used here except they are now extended to writing and solving equations.

Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

Describe: Number of nectarines is three times the number of grapefruit.
Number of nectarines plus number of grapefruit equals 36.

	Define		**Do**	**Decide**
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high
Trial 2:	10	30	40	too high

After several trials to establish a pattern in the problem, you can generalize it using a variable. Since we could try any number of grapefruit, use x to represent it. The pattern for the number of nectarines is three times the number of grapefruit, or $3x$. The total pieces of fruit is the sum of column one and column two, so our table becomes:

Define		**Do**	**Decide**
# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
x	$3x$	$x + 3x$	$= 36$

Since we want the total to agree with the check, our equation is $x + 3x = 36$. Simplifying this yields $4x = 36$, so $x = 9$ (grapefruit) and then $3x = 27$ (nectarines).

Declare: There are 9 grapefruit and 27 nectarines.

Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is 10 feet more than the width, what are the dimensions (length and width) of the rectangle?

Describe/Draw:

width

width + 10
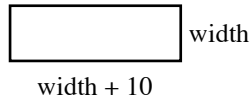
	Define		**Do**	**Decide**
	Width	Length	Perimeter	120?
Trial 1:	10	25	$(10 + 25) \cdot 2 = 70$	too low
Trial 2:	20	30	100	too low

Again, since we could guess any width, we labeled this column x. The pattern for the second column is that it is 10 more than the first: $x + 10$. The perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

Define		**Do**	**Decide**
Width	Length	Perimeter	120?
x	$x + 10$	$(x + x + 10) \cdot 2$	$= 120$

Solving the equation:

$$(x + x + 10) \cdot 2 = 120$$
$$2x + 2x + 20 = 120$$
$$4x + 20 = 120$$
$$4x = 100 \quad \text{So } x = 25 \text{ (width) and } x + 10 = 35 \text{ (length).}$$

Declare: The width is 25 feet and the length is 35 feet.

 Core Connections, Course 2

Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth $2.40. How many dimes and quarters does Jorge have?

Describe: The number of quarters plus 10 equals the number of dimes.
The total value of the coins is $2.40.

	Define				**Do**	**Decide**
	Quarters	Dimes	Value of Quarters	Value of Dimes	Total Value	$2.40?
Trial 1:	10	20	2.50	2.00	4.50	too high
Trial 2:	8	18	2.00	1.80	3.80	too high
	x	$x + 10$	$0.25x$	$0.10(x + 10)$	$0.25x + 0.10(x + 10)$	

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes x, but then in the table the Value of Quarters column is $0.25x$. Thus the number of dimes is $x + 10$, but the value of dimes is $0.10(x + 10)$. Finally, to find the numbers, the equation becomes $0.25x + 0.10(x + 10) = 2.40$.

Solving the equation:
$$0.25x + 0.10x + 1.00 = 2.40$$
$$0.35x + 1.00 = 2.40$$
$$0.35x = 1.40$$
$$x = 4.00$$

Declare: There are 4 quarters worth $1.00 and 14 dimes worth $1.40 for a total value of $2.40.

Problems

Start the problems using the 5-D Process. Then write an equation. Solve the equation.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?

3. Tomás is thinking of a number. If he triples his number and subtracts 13, the result is 305. Of what number is Tomás thinking?

4. Two consecutive numbers have a sum of 123. What are the two numbers?

5. Two consecutive even numbers have a sum of 246. What are the numbers?

6. Joe's age is three times Aaron's age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina's age? Joe's age? Aaron's age?

7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?

8. A wood board 156 centimeters long is cut into three parts. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?

9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate column titles for, "Number of Nickels," "Value of Nickels," "Number of Dimes," and "Value of Dimes.")

10. Tickets to the school play are $5.00 for adults and $3.50 for students. If the total value of all the tickets sold was $2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?

11. A wood board 250 centimeters long is cut into five pieces: three short ones of equal length and two that are both 15 centimeters longer than the shorter ones. What are the lengths of the boards?

12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth $9.60, how many nickels, dimes, and quarters are there?

Answers (Equations may vary.)

1. $x + (x + 26) = 100$
    The lengths of the boards are 37 cm and 63 cm.

2. $x + (x + 5) = 51$
    Thu is 28 years old and her brother is 23 years old.

3. $3x - 13 = 305$
    Tomás is thinking of the number 106.

4. $x + (x + 1) = 123$
    The two consecutive numbers are 61 and 62.

5. $x + (x + 2) = 246$
    The two consecutive even numbers are 142 and 144.

6. $x + (x + 6) + 3(x + 6) = 149$
    Christine is 25, Aaron is 31, and Joe is 93 years old.

7. $2x + 4(38 - x) = 116$
    Farmer Fran has 20 goats and 18 chickens.

8. $x + (x + 15) + (x + 15) = 156$
    The lengths of the boards are 42, 57, and 57 cm.

9. $0.05x + 0.10(15 - x) = 0.90$
    Juan has 12 nickels and 3 dimes.

10. $5x + $3.50(x + 100) = 2517.50$
    There were 255 adult and 355 student tickets purchased for the play.

11. $3x + 2(x + 15) = 250$
    The lengths of the boards are 44 and 59 cm.

12. $0.05x + 0.25x + 0.10(3x) = 9.60$
    Conrad has 16 quarters, 16 nickels, and 48 dimes.

           Core Connections, Course 2

Combining two Expression Mats into an Expression Comparison Mat creates a concrete model for simplifying (and later solving) inequalities and equations.

Tiles may be removed or moved on the mat in the following ways:

(1) Removing the same number of opposite tiles (zeros) on the same side;

(2) Removing an equal number of identical tiles (balanced set) from both the left and right sides;

(3) Adding the same number of opposite tiles (zeros) on the same side; and

(4) Adding an equal number of identical tiles (balanced set) to both the left and right sides.

These strategies are called "legal moves."

After moving and simplifying the Expression Comparison Mat, students are asked to tell which side is greater. Sometimes it is only possible to tell which side is greater if you know possible values of the variable.

Example 1

Determine which side is greater by using legal moves to simplify.
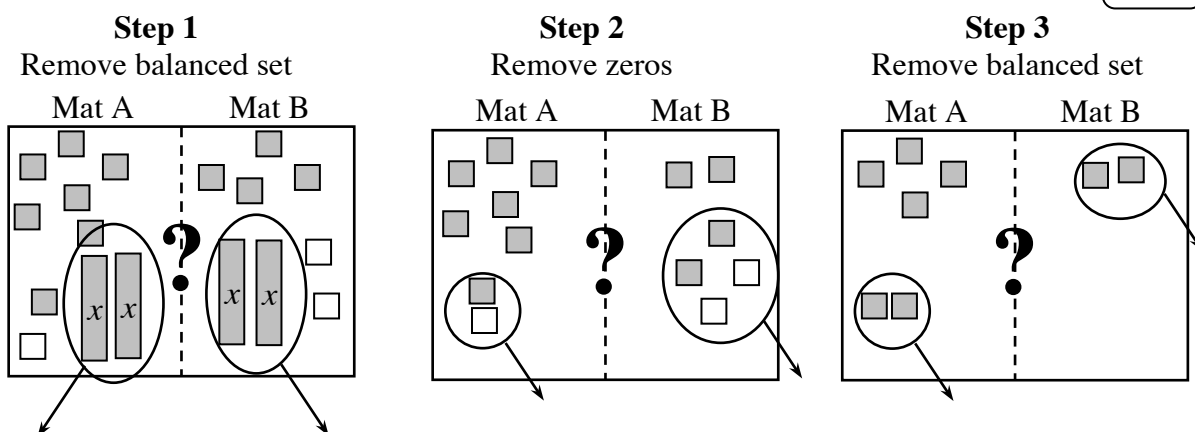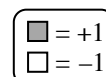
The left side is greater because after Step 3: $4 > 0$. Also, after Step 2: $6 > 2$. Note that this example shows only one of several possible strategies.

Example 2

Use legal moves so that all the *x*-variables are on one side and all the unit tiles are on the other.

Step 1	**Step 2**	**Step 3**
Add balanced set	Add balanced set	Remove zeros
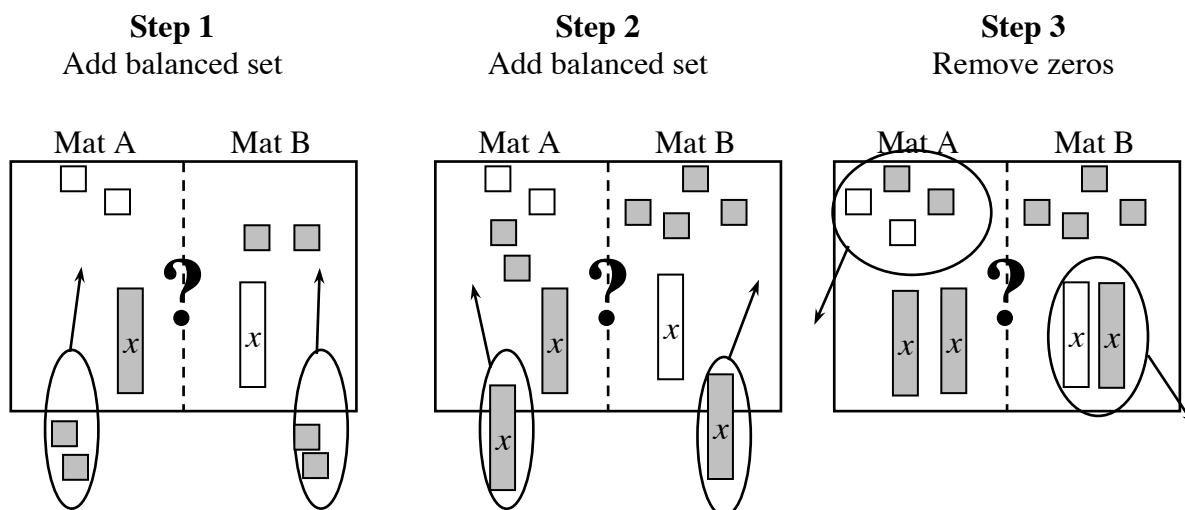
What remains is $2x$ on Mat A and 4 on Mat B. There are other possible arrangements. Whatever the arrangement, it is not possible to tell which side is greater because we do not know the value of "*x*." Students are expected to record the results algebraically as directed by the teacher. One possible recording is shown at right.

Mat A	Mat B
$x - 2$	$-x + 2$
$x - 2 + 2$	$-x + 2 + 2$
$x + x - 2 + 2$	$-x + x + 2 + 2$
$2x$	4

 Core Connections, Course 2

Problems

$\blacksquare$ = +1
$\square$ = –1
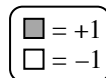
For each of the problems below, use the strategies of removing zeros or simplifying by removing balanced sets to determine which side is greater, if possible. Record your steps.

1.

2.

3.
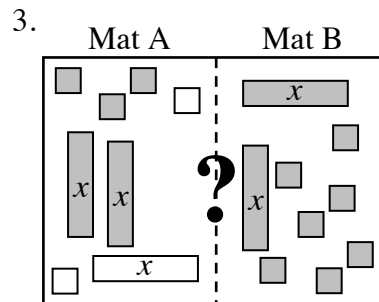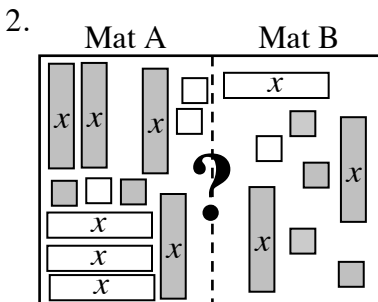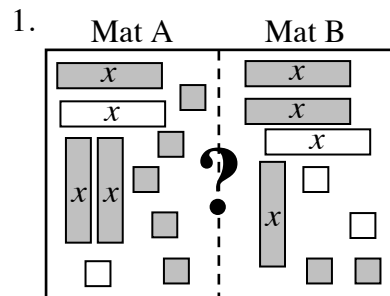
4. Mat A: $5 + (-8)$
    Mat B: $-7 + 6$

5. Mat A: $2(x + 3) - 2$
    Mat B: $4x - 2 - x + 4$

6. Mat A: $4 + (-2x) + 4x$
    Mat B: $x^2 + 2x + 3 - x^2$

For each of the problems below, use the strategies of removing zeros or adding/removing balanced sets so that all the *x*-variables are on one side and the unit tiles are on the other. Record your steps.

7.

8.

9.
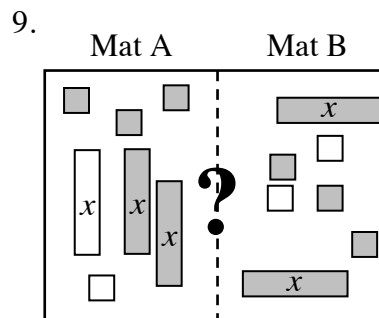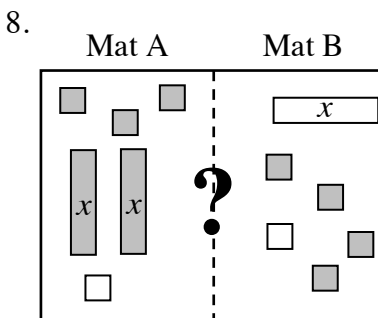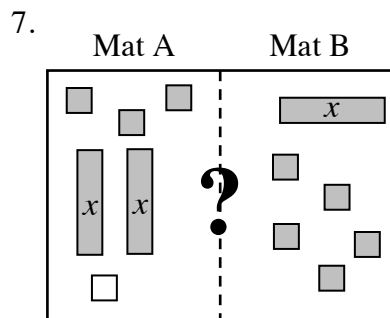
10. Mat A: $3x - 2$
    Mat B: $2x + 1$

11. Mat A: $4x + 2 + (-5)$
    Mat B: $2x + 3 + (-8)$

12. Mat A: $2x + 3$
    Mat B: $-x - 3$

Answers (Answers to problems 7 through 12 may vary.)

1.   A is greater
2.   B is greater
3.   not possible to tell
4.   B is greater
5.   not possible to tell
6.   A is greater
7.   A: *x*; B: 3
8.   A: 3*x*; B: 1
9.   A: 1; B: *x*
10. A: *x*; B: 3
11. A: 2*x*; B: –2
12. A: 3*x*; B: –6

GRAPHING INEQUALITIES

The solutions to an equation can be represented as a point (or points) on the number line. If the expression comparison mat has a range of solutions, the solution is expressed as an inequality represented by a ray or segment with solid or open endpoints. Solid endpoints indicate that the endpoint is included in the solution ($\leq$ or $\geq$), while the open dot indicates that it is not part of the solution ($<$ or $>$).

Example 1

$x > 6$

Example 2

$x \leq -1$

Example 3

$-1 \leq y < 6$

Example 4

$y \geq -2$
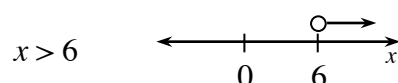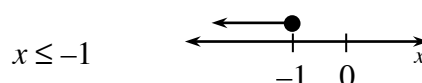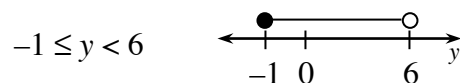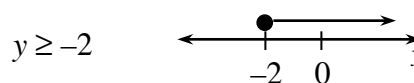
Problems

Graph each inequality on a number line.

1. $m < 2$

2. $x \leq -1$

3. $y \geq 3$

4. $-1 \leq x \leq 3$

5. $-6 < x < -2$

6. $-1 < x \leq 2$

7. $m > -9$

8. $x \neq 1$

9. $x \leq 3$

Answers

1.

2.

3.

4.

5.

6.

7.

8.

9.

       Core Connections, Course 2
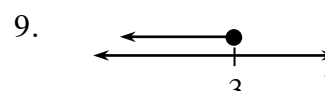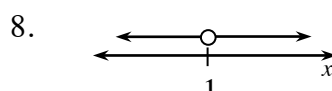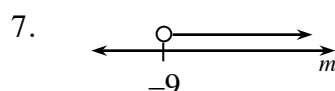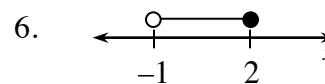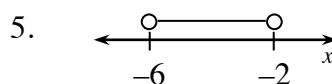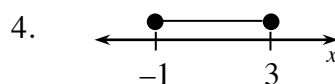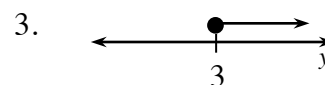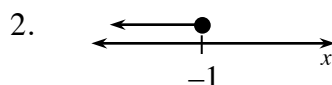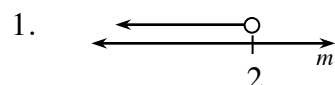
SOLVING INEQUALITIES

To solve an inequality, examine both of the expressions on an expression comparison mat. Use the result as a dividing point on the number line. Then test a value from each side of the dividing point on the number line in the inequality. If the test number is true, then that part of the number line is part of the solution. In addition, if the inequality is ≥ or ≤, then the dividing point is part of the solution and is indicated by a solid dot. If the inequality is > or <, then the dividing point is not part of the solution, indicated by an open dot.

For additional information, see the Math Notes box in Lesson 6.1.4 of the *Core Connections, Course 2* text.

Example 1

$9 \geq m + 2$

Solve the equation: $9 = m + 2$
$7 = m$

Draw a number line. Put a solid dot at 7.
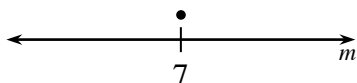
Test a number on each side of 7 in the original inequality. We use 10 and 0.
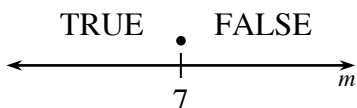
$m = 0$	$m = 10$
$9 > 0 + 2$	$9 > 10 + 2$
$9 > 2$	$9 > 12$
TRUE	FALSE

The solution is $m \leq 7$.
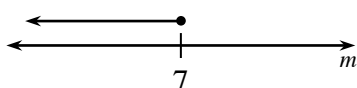
Example 2

$-2x - 3 < x + 6$

Solve the equation: $-2x - 3 = x + 6$
$-2x = x + 9$
$-3x = 9$
$x = -3$

Draw a number line. Put an open dot at –3.
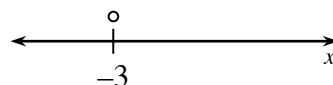
Test 0 and –4 in the original inequality.
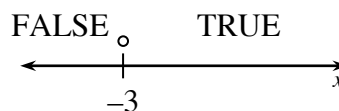
$x = -4$	$x = 0$
$-2(-4) - 3 < -4 + 6$	$-2(0) - 3 < 0 + 6$
$8 - 3 < 2$	$-3 < 6$
$5 < 2$	TRUE
FALSE	

The solution is $x > -3$.
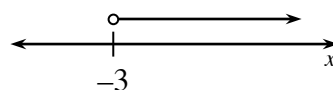
Problems

Solve each inequality.

1. $x + 3 > -1$

2. $y - 3 \leq 5$

3. $-3x \leq -6$

4. $2m + 1 \geq -7$

5. $-7 < -2y + 3$

6. $8 \geq -2m + 2$

7. $2x - 1 < -x + 8$

8. $2(m + 1) \geq m - 3$

9. $3m + 1 \leq m + 7$

Answers

1. $x > -4$

2. $y \leq 8$

3. $x \geq 2$

4. $m \geq -4$

5. $y < 5$

6. $m \geq -3$

7. $x < 3$

8. $m \geq -5$

9. $m \leq 3$

 Core Connections, Course 2

Using a Two-Region Equation Mat

Students combined two Expression Mats to figure out what value(s) of the variable make(s) one expression greater than the other. Now two Expression Mats are combined into an Equation Mat as a concrete model for solving equations. Practicing solving equations using this model will help students transition to solving equations abstractly with better accuracy and understanding.

In general, and as shown in the example below, start by simplifying the Expression Mat. Next, isolate the variables on one side of the Equation Mat and the non-variables (unit tiles) on the other by adding/removing balanced sets and zeros. Then determine the value of the variable. Students are expected to be able to record and explain their steps.

For additional information, see the Math Notes box in Lesson 6.2.1 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 8 materials.

Procedure and Example

Solve: $x + (-4) + 3x = 2x - 1 + 3$

First build the equation on the Equation Mat.

Second, simplify by removing zeros (–1 and +1 on the right side of the mat).

Third, remove a balanced set ($2x$) from both sides.

$$4x - 4 = 2x + 2$$

Isolate the variable by adding a balanced set (+4) to both sides and remove the zeros on the left side.

$$2x - 4 = 2$$
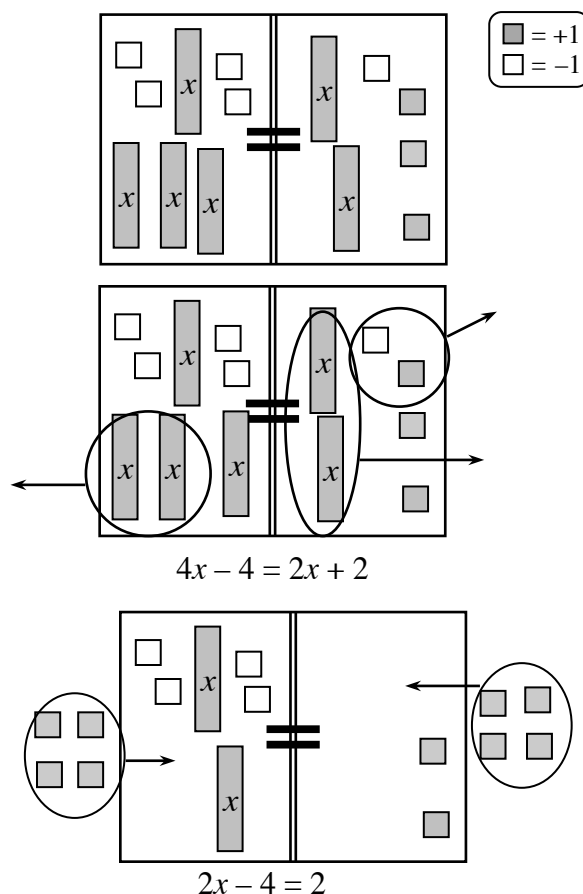
Example continues on next page →

Example continued from previous page.

Finally, since both sides of the equation are equal, determine the value of x by dividing.
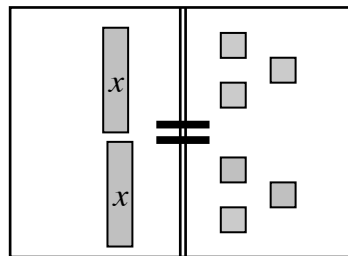
$2x = 6$
$x = 3$

Once the students understand how to solve equations using an Equation Mat, they may use the visual experience of moving the tiles to solve equations with variables and numbers. The procedures for moving variables and numbers in the solving process follow the same rules.

Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example $2 = 4$), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 2 = x + 2$) it means that *all numbers* are solutions. For more information about these special cases, see the Math Notes box in Lesson 6.2.6 of the *Core Connections, Course 2* text.

Example 1

Solve: $3x + 3x - 1 = 4x + 9$

Solution:

$3x + 3x - 1 = 4x + 9$	problem
$6x - 1 = 4x + 9$	simplify by combining like terms
$2x = 10$	add 1, subtract $4x$ on each side
$x = 5$	divide by 2

Example 2

Solve: $-2x + 1 + 3(x - 1) = -4 + -x - 2$

Solution:

$-2x + 1 + 3(x - 1) = -4 + -x - 2$	problem
$-2x + 1 + 3x - 3 = -x - 6$	distribute the 3 on the left side
$x - 2 = -x - 6$	simplify by combining like terms
$2x = -4$	add x, add 2 to each side
$x = -2$	divide by 2

 Core Connections, Course 2

Problems

Solve each equation.

1. $3x + 2 + x = x + 5$ 2. $4x - 2 - 2x = x - 5$

3. $2x - 3 = -x + 3$ 4. $1 + 3x - x = x - 4 + 2x$

5. $4 - 3x = 2x - 6$ 6. $3 + 3x - x + 2 = 3x + 4$

7. $-x - 3 = 2x - 6$ 8. $-4 + 3x - 1 = 2x + 1 + 2x$

9. $-x + 3 = 6$ 10. $5x - 3 + 2x = x + 2 + x$

11. $2x - 7 = -x - 1$ 12. $-2 + 3x = x - 2 - 4x$

13. $-3x + 7 = x - 1$ 14. $1 + 2x - 4 = -3 + x$

15. $3(x + 2) = x + 2$ 16. $2(x - 2) + x = 5$

17. $10 = x + 5 + x$ 18. $-x + 2 = x - 5 - 3x$

19. $3(4 + x) = x + 6$ 20. $6 - x - 3 = 4(x - 2)$

Answers

1.	1	2.	-3	3.	2	4.	5	5.	2
6.	1	7.	1	8.	-6	9.	-3	10.	1
11.	2	12.	0	13.	2	14.	0	15.	-2
16.	3	17.	$2\frac{1}{2}$	18.	-7	19.	-3	20.	$2\frac{1}{5}$

SOLVING EQUATIONS IN CONTEXT 6.2.4 – 6.2.7

Initially, equations are solved either by applying math facts (for example, $4x = 12$, since $4 \cdot 3 = 12$, $x = 3$) or by matching equal quantities, simplifying the equation, and using math facts as shown in the examples below. Equations are often written in the context of a geometric situation.

Write an equation that represents each situation and find the value of the variable.

Example 1

$$x + 10 = 32$$
$$x = 22$$

Example 2
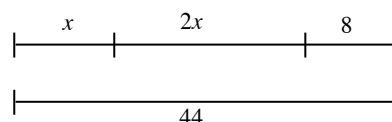
$$x + 2x + 8 = 44$$
$$x + 2x = 36$$
$$3x = 36$$
$$x = 12$$

Example 3
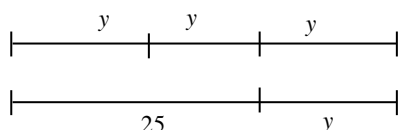
$$3y = 25 + y$$
$$2y = 25$$
$$y = 12.5$$

Example 4
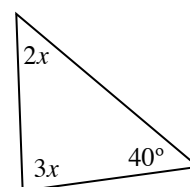
$$2x + 3x + 40 = 180$$
$$2x + 3x = 140$$
$$5x = 140$$
$$x = 18$$

 Core Connections, Course 2

Problems

Write an equation that represents each situation and then find the value of the variable.

1.
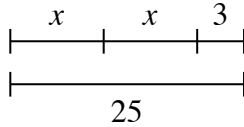
2.
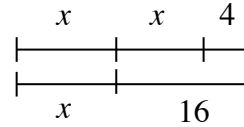
3.
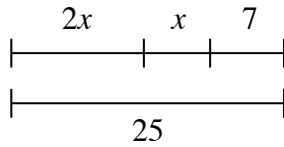
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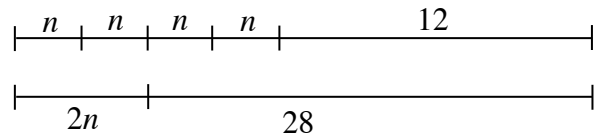
5.
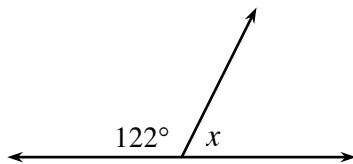
6.
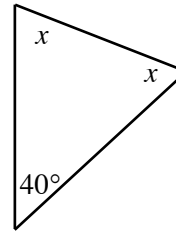
Solve each equation.

7. $x + 7 = -9$

8. $y - 2 = -3$

9. $-3y = 24$

10. $\frac{m}{2} = -6$

11. $3x + 2 = 11$

12. $4x + x + 5 = 25$

13. $m + 2m + 7 = m + 11$

14. $x + 9 + x + x = 30$

15. $3 - y = 9$

16. $4k + 1 = -7$

17. $x + 3x + x + 7 = 52$

18. $3m + 7 = m + 11$

19. $2(y + 3) = -12$

20. $3(c + 2) + c + 1 = 57$

Answers

1. $2x + 3 = 25$; $x = 11$

2. $2x + 4 = x + 16$; $x = 12$

3. $3x + 7 = 25$; $x = 6$

4. $4n + 12 = 2n + 28$; $n = 8$

5. $122 + x = 180$; $x = 58°$

6. $2x + 40 = 180$; $x = 70°$

7. $x = -16$

8. $y = -1$

9. $y = -8$

10. $m = -12$

11. $x = 3$

12. $x = 4$

13. $m = 2$

14. $x = 7$

15. $y = -6$

16. $k = -2$

17. $x = 9$

18. $m = 2$

19. $y = -9$

20. $c = 12.5$

     Core Connections, Course 2

Distance (*d*) equals the product of the rate of speed (r) and the time (*t*). This relationship is shown below in three forms:

$$d = r \cdot t \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

It is important that the units of measure are consistent.

Example 1

Find the rate of speed of a passenger car if the distance traveled is 572 miles and the time elapsed is 11 hours.

$$572 \text{ miles} = r \cdot 11 \text{ hours} \implies \frac{572 \text{ miles}}{11 \text{ hours}} = r \implies 52 \text{ miles/hour} = \text{rate}$$

Example 2

Find the distance traveled by a train at 135 miles per hour for 40 minutes.

The units of time are not the same so we need to change 40 minutes into hours. $\frac{40}{60} = \frac{2}{3}$ hour.

$$d = (135 \text{ miles/hour})(\tfrac{2}{3} \text{ hour}) \implies d = 90 \text{ miles}$$

Example 3

The Central Middle School hamster race is fast approaching. Fred said that his hamster traveled 60 feet in 90 seconds and Wilma said she timed for one minute and her hamster traveled 12 yards. Which hamster has the fastest rate?

rate $= \frac{\text{distance}}{\text{time}}$ but all the measurements need to be in the same units. In this example, we use feet and minutes.

Fred's hamster: rate $= \frac{60 \text{ feet}}{1.5 \text{ minutes}} \implies$ rate = 40 feet/minute

Wilma's hamster: rate $= \frac{36 \text{ feet}}{1 \text{ minute}} \implies$ rate = 36 feet/minute

Fred's hamster is faster.

Problems

Solve the following problems.

1. Find the time if the distance is 157.5 miles and the speed is 63 mph.

2. Find the distance if the speed is 67 mph and the time is 3.5 hours.

3. Find the rate if the distance is 247 miles and the time is 3.8 hours.

4. Find the distance if the speed is 60 mph and the time is 1 hour and 45 minutes.

5. Find the rate in mph if the distance is 3.5 miles and the time is 20 minutes.

6. Find the time in minutes if the distance is 2 miles and the rate is 30 mph.

7. Which rate is faster? A: 60 feet in 90 seconds or B: 60 inches in 5 seconds

8. Which distance is longer? A: 4 feet/second for a minute or B: 3 inches/min for an hour

9. Which time is shorter? A: 4 miles at 60 mph or B: 6 miles at 80 mph

Answers

1. 2.5 hr   2. 234.5 mi   3. 65 mph   4. 105 miles   5. 10.5 mph

6. 4 min   7. B   8. A   9. A

 Core Connections, Course 2

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient. Recall that a reciprocal is the multiplicative inverse of a number, that is, the product of the two numbers is 1. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, $\frac{1}{2}$ is $\frac{2}{1}$, and 5 is $\frac{1}{5}$.

Scaling may also be used with percentage problems where a quantity is increased or decreased by a certain percent. Scaling by a factor of 1 does not change the quantity. Increasing by a certain percent may be found by multiplying by (1 + the percent) and decreasing by a certain percent may be found by multiplying by (1 – the percent).

For additional information, see the Math Notes box in Lesson 7.1.4 of the *Core Connections, Course 2* text.

Example 1

The large triangle at right was reduced by a scale factor of $\frac{2}{5}$ to create a similar triangle. If the side labeled x now has a length of 80' in the new figure, what was the original length?

To undo the reduction, multiply 80' by the reciprocal of $\frac{2}{5}$, namely $\frac{5}{2}$, or divide 80' by $\frac{2}{5}$.

$80' \div \frac{2}{5}$ is the same as $80' \cdot \frac{5}{2}$, so $x = 200'$.
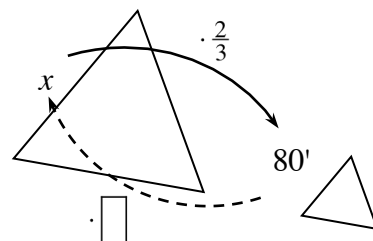
Example 2

Solve: $\frac{2}{3} x = 12$

Method 1: Use division and a Giant One

$$\frac{2}{3} x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3} x = 12$$

$$\frac{3}{2}\left(\frac{2}{3} x\right) = \frac{3}{2}(12)$$

$$x = 18$$

Example 3

Samantha wants to leave a 15% tip on her lunch bill of $12.50. What scale factor should be used and how much money should she leave?

Since tipping increases the total, the scale factor is $(1 + 15\%) = 1.15$.
She should leave $(1.15)(12.50) = \$14.38$ or about $14.50.

Example 4

Carlos sees that all DVDs are on sales at 40% off. If the regular price of a DVD is $24.95, what is the scale factor and how much is the sale price?

If items are reduced 40%, the scale factor is $(1 - 40\%) = 0.60$.
The sale price is $(0.60)(24.95) = \$14.97$.

Problems

1. A rectangle was enlarged by a scale factor of $\frac{5}{2}$ and the new width is 40 cm. What was the original width?

2. A side of a triangle was reduced by a scale factor of $\frac{2}{3}$. If the new side is now 18 inches, what was the original side?

3. The scale factor used to create the design for a backyard is 2 inches for every 75 feet ($\frac{2}{75}$). If on the design, the fire pit is 6 inches away from the house, how far from the house, in feet, should the fire pit be dug?

4. After a very successful year, Cheap-Rentals raised salaries by a scale factor of $\frac{11}{10}$. If Luan now makes $14.30 per hour, what did she earn before?

5. Solve: $\frac{3}{4}x = 60$

6. Solve: $\frac{2}{5}x = 42$

7. Solve: $\frac{3}{5}y = 40$

8. Solve: $-\frac{8}{3}m = 6$

9. What is the total cost of a $39.50 family dinner after you add a 20% tip?

10. If the current cost to attend Magicland Park is now $29.50 per person, what will be the cost after a 8% increase?

11. Winter coats are on clearance at 60% off. If the regular price is $79, what is the sale price?

12. The company president has offered to reduce his salary 10% to cut expenses. If she now earns $175,000, what will be her new salary?

 Core Connections, Course 2

Answers

1. 16 cm

2. 27 inches

3. 225 feet

4. $13.00

5. 80

6. 105

7. $66\frac{2}{3}$

8. $-2\frac{1}{4}$

9. $47.40

10. $31.86

11. $31.60

12. $157,500

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving one-step equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient.

To remove fractions in more complicated equations students use "Fraction Busters." Multiplying all of the terms of an equation by the common denominator will remove all of the fractions from the equation. Then the equation can be solved in the usual way.

For additional information, see the Math Notes box in Lesson 7.1.6 of the *Core Connections, Course 2* text.

Example of a One-Step Equation

Solve: $\frac{2}{3}x = 12$

Method 1: Use division and common denominators

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3}x = 12$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(12)$$

$$x = 18$$

Example of Fraction Busters

Solve: $\frac{x}{2} + \frac{x}{5} = 6$

Multiplying by 10 (the common denominator) will eliminate the fractions.

$$10\left(\frac{x}{2} + \frac{x}{5}\right) = 10(6)$$

$$10\left(\frac{x}{2}\right) + 10\left(\frac{x}{5}\right) = 10(6)$$

$$5x + 2x = 60$$

$$7x = 60 \Rightarrow x = \frac{60}{7} \approx 8.57$$

 Core Connections, Course 2

Problems

Solve each equation.

1. $\frac{3}{4}x = 60$

2. $\frac{2}{5}x = 42$

3. $\frac{3}{5}y = 40$

4. $-\frac{8}{3}m = 6$

5. $\frac{3x+1}{2} = 5$

6. $\frac{x}{3} - \frac{x}{5} = 3$

7. $\frac{y+7}{3} = \frac{y}{5}$

8. $\frac{m}{3} - \frac{2m}{5} = \frac{1}{5}$

9. $-\frac{3}{5}x = \frac{2}{3}$

10. $\frac{x}{2} + \frac{x-3}{5} = 3$

11. $\frac{1}{3}x + \frac{1}{4}x = 4$

12. $\frac{2x}{5} + \frac{x-1}{3} = 4$

Answers

1. $x = 80$

2. $x = 105$

3. $y = 66\frac{2}{3}$

4. $m = -\frac{9}{4}$

5. $y = 3$

6. $x = 22.5$

7. $y = -17\frac{1}{2}$

8. $m = -3$

9. $x = -\frac{10}{9}$

10. $x = \frac{36}{7}$

11. $x = \frac{48}{7}$

12. $x = \frac{65}{11}$

A percent increase is the amount that a quantity has increased based on a percent of the original amount. A percent decrease is the amount that a quantity has decreased based on a percent of the original amount. An equation that represents either situation is:

amount of increase or decrease = (% change)(original amount)

For additional information see the Math Notes box in Lesson 7.1.1 of the *Core Connections, Course 2* text.

Example 1

A town's population grew from 1879 to 7426 over five years. What was the percent increase in the population?

- Subtract to find the change:

$$7426 - 1879 = 5547$$

- Put the known numbers in the equation:

$$5547 = (x)(1879)$$

- The scale factor becomes x, the unknown:

$$\frac{5547}{1879} = x$$

- Divide: $x = \frac{5547}{1879} \approx 2.952$

- Change to percent: $x = 295.2\%$

The population increased by 295.2%.

Example 2

A sumo wrestler retired from sumo wrestling and went on a diet. When he retired he weighed 385 pounds. After two years he weighed 238 pounds. What was the percent decrease in his weight?

- Subtract to find the change:

$$385 - 238 = 147$$

- Put the known numbers in the equation:

$$147 = (x)(385)$$

- The scale factor becomes x, the unknown:

$$\frac{147}{385} = x$$

- Divide: $x = \frac{147}{385} \approx 0.382$

- Change to percent: $x \approx 38.2\%$

His weight decreased by about 38.2%.

 Core Connections, Course 2

Problems

Solve the following problems.

1. Forty years ago gasoline cost $0.30 per gallon on average. Ten years ago gasoline averaged about $1.50 per gallon. What is the percent increase in the cost of gasoline?

2. When Spencer was 5, he was 28 inches tall. Today he is 5 feet 3 inches tall. What is the percent increase in Spencer's height?

3. The cars of the early 1900s cost $500. Today a new car costs an average of $27,000. What is the percent increase of the cost of an automobile?

4. The population of the U.S. at the first census in 1790 was 3,929 people. By 2000 the population had increased to 284,000,000! What is the percent increase in the population?

5. In 2000 the rate for a first class U.S. postage stamp increased to $0.34. This represents a $0.31 increase since 1917. What is the percent increase in cost since 1917?

6. In 1906 Americans consumed an average of 26.85 gallons of whole milk per year. By 1998 the average consumption was 8.32 gallons. What is the percent decrease in consumption of whole milk?

7. In 1984 there were 125 students for each computer in U.S. public schools. By 1998 there were 6.1 students for each computer. What is the percent decrease in the ratio of students to computers?

8. Sara bought a dress on sale for $30. She saved 45%. What was the original cost?

9. Pat was shopping and found a jacket with the original price of $120 on sale for $9.99. What was the percent decrease in the cost?

10. The price of a pair of pants decreased from $49.99 to $19.95. What was the percent decrease in the price?

Answers

1.	400%	2.	125%	3.	5300%	4.	7,228,202.4%
5.	≈ 1033.33%	6.	69.01%	7.	95.12%	8.	$55
9.	91.7%	10.	60.1%				

In Course 2 students are introduced to simple interest, the interest is paid only on the original amount invested. The formula for simple interest is: $I = Prt$ and the total amount including interest would be: $A = P + I$.

For additional information, see the Math Notes box in Lesson 7.1.8 of the *Core Connections, Course 2* text.

Example

Wayne earns 5.3% simple interest for 5 years on $3000. How much interest does he earn and what is the total amount in the account?

Put the numbers in the formula $I = Prt$.	$I = 3000(5.3\%)5$
Change the percent to a decimal.	$= 3000(0.053)5$
Multiply.	$= 795$ Wayne would earn $795 interest.
Add principal and interest.	$3000 + $795 = $3795 in the account

Problems

Solve the following problems.

1. Tong loaned Jody $50 for a month. He charged 5% simple interest for the month. How much did Jody have to pay Tong?

2. Jessica's grandparents gave her $2000 for college to put in a savings account until she starts college in four years. Her grandparents agreed to pay her an additional 7.5% simple interest on the $2000 for every year. How much extra money will her grandparents give her at the end of four years?

3. David read an ad offering $8\frac{3}{4}\%$ simple interest on accounts over $500 left for a minimum of 5 years. He has $500 and thinks this sounds like a great deal. How much money will he earn in the 5 years?

4. Javier's parents set an amount of money aside when he was born. They earned 4.5% simple interest on that money each year. When Javier was 15, the account had a total of $1012.50 interest paid on it. How much did Javier's parents set aside when he was born?

5. Kristina received $125 for her birthday. Her parents offered to pay her 3.5% simple interest per year if she would save it for at least one year. How much interest could Kristina earn?

Answers

1. $I = 50(0.05)1 = \$2.50$; Jody paid back $52.50.

2. $I = 2000(0.075)4 = \$600$

3. $I = \$500(0.0875)5 = \$218.75$

4. $\$1012.50 = x(0.045)15$; $x = \$1500$

5. $I = 125(0.035)1 = \$4.38$

Students represent distributions of single-variable data numerical data using dot plots, stem-and-leaf plots, box plots, and histograms. They represent categorical one-variable data on bar graphs. Each representation communicates information in a slightly different way.

STEM-AND-LEAF-PLOTS

A stem-and-leaf plot is a way to display data that shows the individual values from a set of data and how the values are distributed. The "stem" part on the graph represents all of the digits except the last one. The "leaf" part of the graph represents the last digit of each number.

Read more about stem-and-leaf plots, and how they compare to dot plots and histograms, in the Math Notes box in Lesson 7.1.1 of the *Core Connections, Course 2* text.

Example 1

Make a stem-and-leaf plot of this set of data: 34, 31, 37, 44, 38, 29, 34, 42, 43, 34, 52, and 41.

```
2 | 9
3 | 1 4 4 4 7 8
4 | 1 2 3 4
5 | 2
```

Example 2

Make a stem-and-leaf plot of this set of data: 392, 382, 380, 392, 378, 375, 395, 377, and 377.

```
37 | 5 7 7 8
38 | 0 2
39 | 2 2 5
```

Problems

Make a stem-and-leaf plot of each set of data.

1. 29, 28, 34, 30, 33, 26, 18, and 34.

2. 25, 34, 27, 25, 19, 31, 42, and 30.

3. 80, 89, 79, 84, 95, 79, 89, 67, 82, 76, 92, 89, 81, and 123.

4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

Answers

1.

1	8
2	6 8 9
3	0 3 4 4

2.

1	9
2	5 5 7
3	0 1 4 5
4	2

3.

6	7
7	6 9 9
8	0 1 2 4 9 9 9
9	2 5
10	
11	
12	3

4.

9	1
10	0 1 1 3 4 5 7 8 9
11	1 3 3 6 8

A quadrilateral is any four-sided polygon. There are six special cases of quadrilaterals with which students should be familiar.

- Trapezoid – A quadrilateral with at least one pair of parallel sides.

- Parallelogram – A quadrilateral with both pairs of opposite sides parallel.

- Rectangle – A quadrilateral with four right angles.

- Rhombus – A quadrilateral with four sides of equal length.

- Square – A quadrilateral with four right angles *and* four sides of equal length.

- Kite – A quadrilateral with two distinct pairs of consecutive sides of equal length.
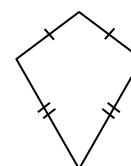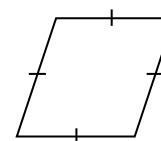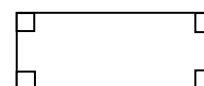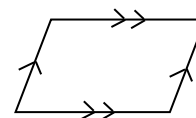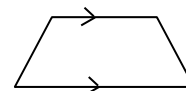
Names of Basic Angles

Acute angles are angles with measures between (but not including) 0° and 90°, right angles measure 90°, and obtuse angles measure between (but not including) 90° and 180°. A straight angle measures 180°.

37°	90°	130°	180°
acute	right	obtuse	straight
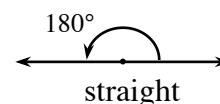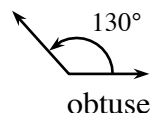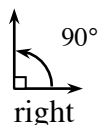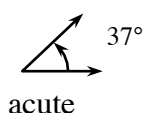
Students use protractors to measure the size of angles.

For more information see the Math Notes box in Lesson 8.3.1 of the *Core Connections, Course 2* text.

Example 1

For the figure at right, describe the quadrilateral using *all* terms that are appropriate.

It is a rectangle since it has four right angles.

It is a parallelogram since it has two pairs of parallel sides.

It is also a trapezoid since it has at least one pair of parallel sides.
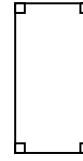
Example 2

Describe the angle at right as right, obtuse or acute. Estimate the size of the angle and then use a protractor to measure the angle. It may be necessary to trace the angle and extend the sides.

This angle opens narrower than a right angle so it is acute. Using a protractor shows that the angle measures 60°.
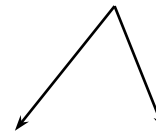
Problems

For each figure, describe the quadrilateral using *all* terms that are appropriate. Assume that sides that look parallel are parallel.

1.
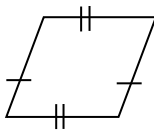
2.
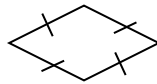
3.
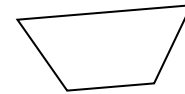
4.
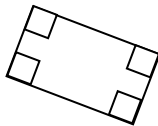
5.
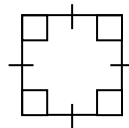
6.
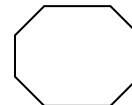
Describe the angles below as right, obtuse or acute. Estimate the size of the angle and then use a protractor to measure the angle.

7.
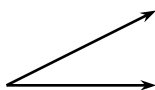
8.

9.
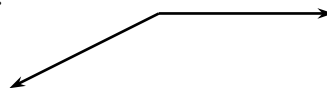
10.

11.
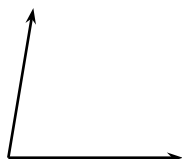
12.
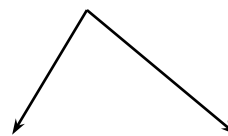
Answers

1.	parallelogram, trapezoid	2.	rhombus, parallelogram, trapezoid	3.	trapezoid
4.	rectangle, parallelogram, trapezoid	5.	square, rhombus, rectangle, parallelogram, trapezoid	6.	not a quadrilateral
7.	acute, 25°	8.	obtuse, 110°	9.	obtuse, 153°
10.	right, 90°	11.	acute, 80°	12.	acute, 82°

      Core Connections, Course 2

Properties of Angle Pairs

Intersecting lines form four angles. The pairs of angles across from each other are called vertical angles. The measures of vertical angles are equal.
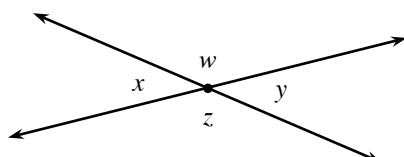
$\angle x$ and $\angle y$ are vertical angles
$\angle w$ and $\angle z$ are vertical angles

If the sum of the measures of two angles is exactly 180°, then the angles are called supplementary angles.
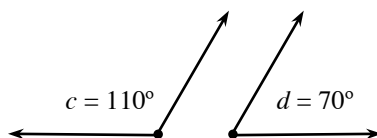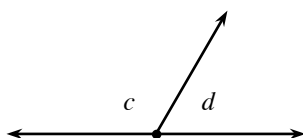
c d $c = 110°$ $d = 70°$ $\angle c$ and $\angle d$ are supplementary angles

If the sum of the measures of two angles is exactly 90°, then the angles are called complementary angles.
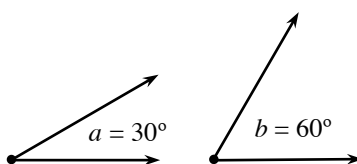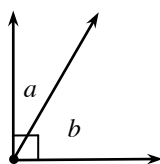
a b $a = 30°$ $b = 60°$ $\angle a$ and $\angle b$ are complementary angles

Angles that share a vertex and one side but have no common interior points (that is, do not overlap each other) are called adjacent angles.
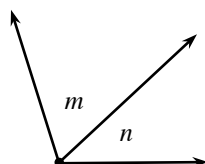
m n $\angle m$ and $\angle n$ are adjacent angles

For additional information, see the Math Notes box in Lesson 8.3.2 of the *Core Connections, Course 2* text.

Example 1

Find the measure of the missing angles if $m\angle 3 = 50°$.
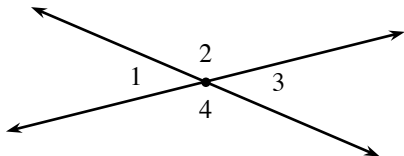
- $m\angle 1 = m\angle 3$ (vertical angles)
  $\Rightarrow m\angle 1 = 50°$

- $\angle 2$ and $\angle 3$ (supplementary angles)
  $\Rightarrow m\angle 2 = 180° - 50° = 130°$

- $m\angle 2 = m\angle 4$ (vertical angles)
  $\Rightarrow m\angle 4 = 130°$

Example 2

Classify each pair of angles below as vertical, supplementary, complementary, or adjacent.
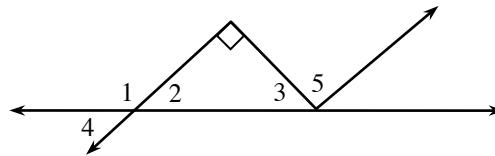
a. $\angle 1$ and $\angle 2$ are adjacent and supplementary

b. $\angle 2$ and $\angle 3$ are complementary

c. $\angle 3$ and $\angle 5$ are adjacent

d. $\angle 1$ and $\angle 4$ are adjacent and supplementary

e. $\angle 2$ and $\angle 4$ are vertical

Problems

Find the measure of each angle labeled with a variable.

1.
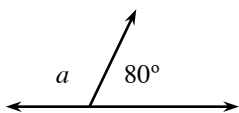
2.
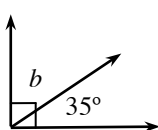
3.
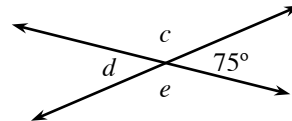
4.
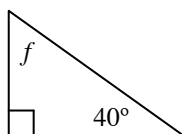
5.
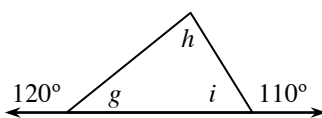
6.
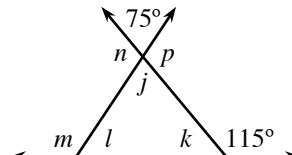
Answers

1. $m\angle a = 100°$

2. $m\angle b = 55°$

3. $m\angle c = 105°$
    $m\angle d = 75°$
    $m\angle e = 105°$

4. $m\angle f = 50°$

5. $m\angle g = 60°$
    $m\angle h = 50°$
    $m\angle i = 70°$

6. $m\angle j = 75°$ $m\angle k = 65°$
    $m\angle l = 40°$ $m\angle m = 140°$
    $m\angle n = 105°$ $m\angle p = 105°$

   Core Connections, Course 2

AREA OF A CIRCLE

In class, students have done explorations with circles and circular objects to discover the relationship between circumference, diameter, and pi (π). To read more about the in-class exploration of area, see problems 9-22 through 9-26 (especially 9-26) in the *Core Connections, Course 2* text.

In order to find the area of a circle, students need to identify the radius of the circle. The radius is half the diameter. Next they will square the radius and multiply the result by π. Depending on the teacher's or book's preference, students may use $\frac{22}{7}$ for π when the radius or diameter is a fraction, 3.14 for π as an approximation, or the π button on a calculator. When using the π button, most teachers will want students to round to the nearest tenth or hundredth.

The formula for the area of a circle is: $A = r^2\pi$.

Example 1

Find the area of a circle with $r = 17$ feet.

$A = (17)^2\pi$
$\approx (17 \cdot 17)(3.14)$
≈ 907.46 square feet

Example 2

Find the area of a circle with $d = 84$ cm.

$r = 42$ cm
$A = (42)^2\pi$
$\approx (42 \cdot 42)(3.14)$
≈ 5538.96 square cm

Problems

Find the areas of the circles with the following radii or diameter lengths. Use 3.14 for the value of π. Round to the nearest hundredth.

1. $r = 6$ cm
2. $r = 3.2$ in.
3. $d = 16$ ft
4. $r = \frac{1}{2}$ m

5. $d = \frac{4}{5}$ cm
6. $r = 5$ in.
7. $r = 3.6$ cm
8. $r = 2\frac{1}{4}$ in.

9. $d = 14.5$ ft
10. $r = 12.02$ m

Answers

1. 113.04 cm^2
2. 32.15 in.^{2}
3. 200.96 ft^2
4. $\frac{11}{14}$ m^2

5. $\frac{88}{175}$ cm^2
6. 78.5 in.^{2}
7. 40.69 cm^2
8. $15\frac{51}{56}$ or 15.90 in.^{2}

9. 165.05 ft^2
10. 453.67 m^2

CIRCUMFERENCE

The radius of a circle is a line segment from its center to any point on the circle. The term is also used for the length of these segments. More than one radius are called radii. A chord of a circle is a line segment joining any two points on a circle. A diameter of a circle is a chord that goes through its center. The term is also used for the length of these chords. The length of a diameter is twice the length of a radius.
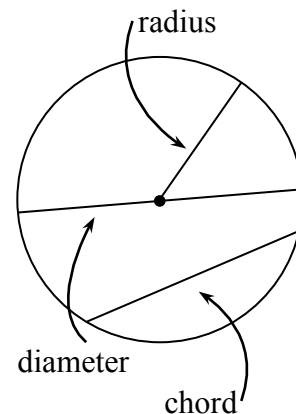
The circumference of a circle is similar to the perimeter of a polygon. The circumference is the length of a circle. The circumference would tell you how much string it would take to go around a circle once.

Circumference is explored by investigating the ratio of the circumference to the diameter of a circle. This ratio is a constant number, pi (π). Circumference is then found by multiplying π by the diameter. Students may use $\frac{22}{7}$, 3.14, or the π button on their calculator, depending on the teacher's or the book's directions.

$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

For additional information, see the Math Notes boxes in Lessons 8.3.3 and 9.1.2 of the *Core Connections, Course 2* text.

Example 1

Find the circumference of a circle with a diameter of 5 inches.

$d = 5$ inches

$C = \pi d$
$ = \pi(5)$
$ \approx 3.14(5)$
$ \approx 15.7$ inches

Example 2

Find the circumference of a circle with a radius of 10 units.

$r = 10$, so $d = 2(10) = 20$

$C \approx 3.14(20)$
$ \approx 62.8$ units

Example 3

Find the diameter of a circle with a circumference of 163.28 inches.

$C = \pi d$
$163.28 = \pi d$
$163.28 \approx 3.14d$
$d \approx \frac{163.28}{3.14}$

$ \approx 52$ inches

 Core Connections, Course 2

Problems

Find the circumference of each circle given the following radius or diameter lengths. Round your answer to the nearest hundredth.

1. $d = 12$ in.   2. $d = 3.4$ cm   3. $r = 2.1$ ft   4. $d = 25$ m   5. $r = 1.54$ mi

Find the circumference of each circle shown below. Round your answer to the nearest hundredth.

6.
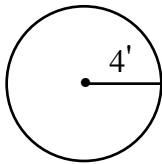
7.
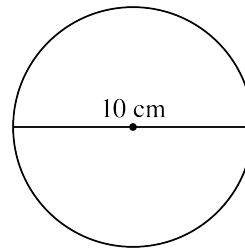
Find the diameter of each circle given the circumference. Round your answer to the nearest tenth.

8. $C = 48.36$ yds   9. $C = 35.6$ ft   10. $C = 194.68$ mm

Answers

1.   37.68 in.   2.   10.68 cm   3.   13.19 ft   4.   78.5 m

5.   9.67 mi   6.   25.12 ft   7.   31.40 cm   8.   15.4 yards

9.   11.3 ft   10. 62 mm

Area is the number of non-overlapping square units needed to cover the interior region of a two-dimensional figure or the surface area of a three-dimensional figure. For example, area is the region that is covered by floor tile (two-dimensional) or paint on a box or a ball (three-dimensional).

For additional information about specific shapes, see the boxes below. For additional general information, see the Math Notes box in Lesson 1.1.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 1 materials.

AREA OF A RECTANGLE

To find the area of a rectangle, follow the steps below.

1. Identify the base.

2. Identify the height.

3. Multiply the base times the height to find the area in square units: $A = bh$.

A square is a rectangle in which the base and height are of equal length. Find the area of a square by multiplying the base times itself: $A = b^2$.

Example
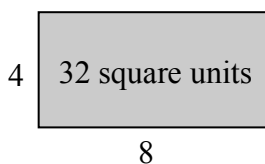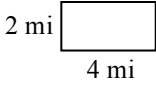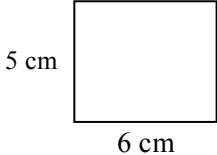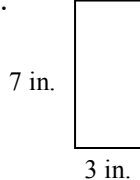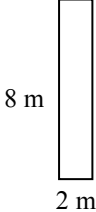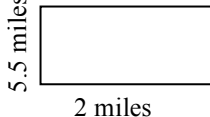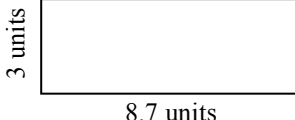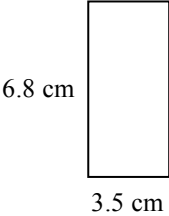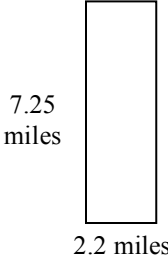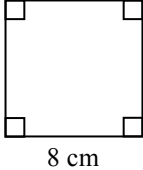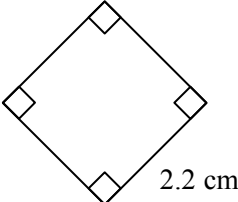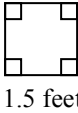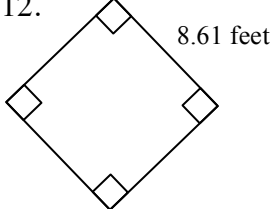
base = 8 units

height = 4 units

$A = 8 \cdot 4 = 32$ square units

Core Connections, Course 2

Problems

Find the areas of the rectangles (figures 1 to 8) and squares (figures 9 to 12) below.

1.

2 mi | 4 mi

2.

5 cm | 6 cm

3.

7 in. | 3 in.

4.

8 m | 2 m

5.

5.5 miles | 2 miles

6.

3 units | 8.7 units

7.

6.8 cm | 3.5 cm

8.

7.25 miles | 2.2 miles

9.

8 cm

10.

2.2 cm

11.

1.5 feet

12.

8.61 feet

Answers

1.	8 sq. miles	2.	30 sq. cm	3.	21 sq. in.	4.	16 sq. m
5.	11 sq. miles	6.	26.1 sq. feet	7.	23.8 sq. cm	8.	15.95 sq. miles
9.	64 sq. cm	10.	4.84 sq. cm	11.	2.25 sq. feet	12.	73.96 sq. feet

AREA OF A PARALLELOGRAM

A parallelogram is easily changed to a rectangle by separating a triangle from one end of the parallelogram and moving it to the other end as shown in the three figures below.
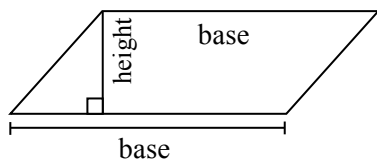
| parallelogram | move triangle | rectangle |
| Step 1 | Step 2 | Step 3 |
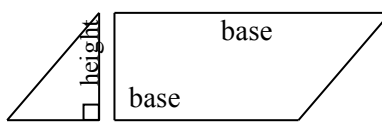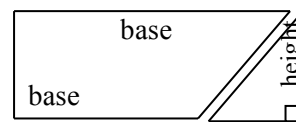
To find the area of a parallelogram, multiply the base times the height as you did with the rectangle: $A = bh$.

Example
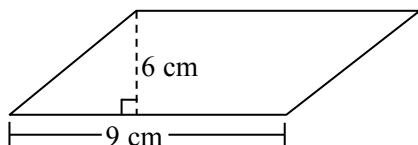
base = 9 cm

height = 6 cm

$A = 9 \cdot 6 = 54$ square cm

Problems

Find the area of each parallelogram below.

1.

6 feet
8 feet

2.

8 cm
10 cm

3.

4 m
11 m

4.

3 cm
13 cm

5.

7.5 in.
12 in.

6.

11.2 ft
15 ft

7.

9.8 cm
11.3 cm

8.

8.4 cm
15.7 cm

 Core Connections, Course 2
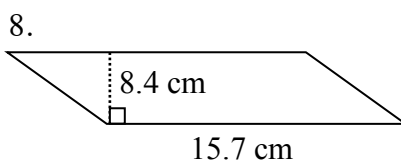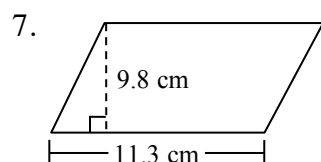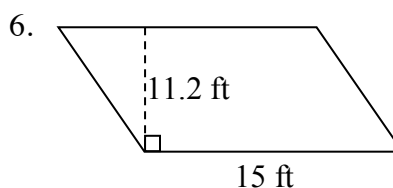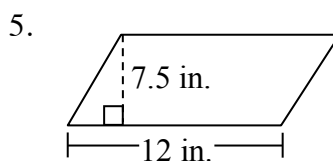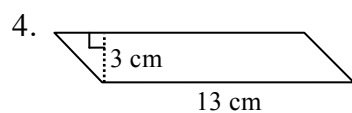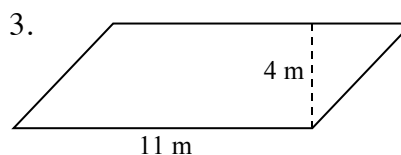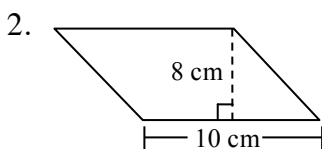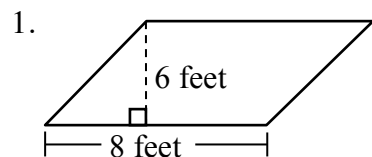
Answers

1. 48 sq. ft   2. 80 sq. cm   3. 44 sq. m   4. 39 sq. cm
5. 90 sq. in.   6. 168 sq. ft   7. 110.74 sq. cm   8. 131.88 sq. cm

AREA OF A TRAPEZOID

A trapezoid is another shape that can be transformed into a parallelogram. Change a trapezoid into a parallelogram by following the three steps below.
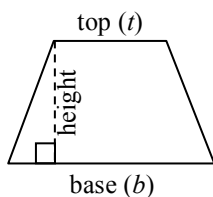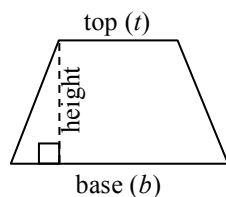
Trapezoid	duplicate the trapezoid and rotate	put the two trapezoids together to form a parallelogram
Step 1	Step 2	Step 3
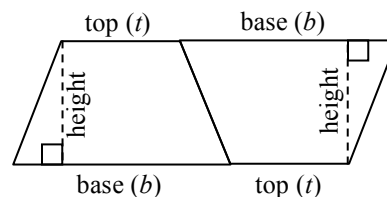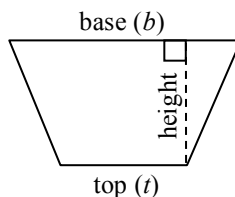
To find the area of a trapezoid, multiply the base of the large parallelogram in Step 3 (base and top) times the height and then take half of the total area. Remember to add the lengths of the base and the top of the trapezoid before multiplying by the height. Note that some texts call the top length the upper base and the base the lower base.

$$A = \tfrac{1}{2}(b+t)h \quad \text{or} \quad A = \tfrac{b+t}{2} \cdot h$$

Example
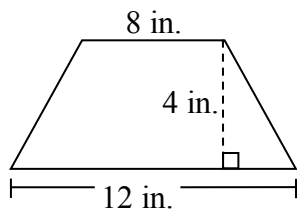
top = 8 in.

base = 12 in.

height = 4 in.

$$A = \tfrac{8+12}{2} \cdot 4 = \tfrac{20}{2} \cdot 4 = 10 \cdot 4 = 40 \text{ in.}^2$$

Problems

Find the areas of the trapezoids below.

1.

3 cm

1 cm

5 cm

2.

10 in.

8 in.

15 in.

3.

2 feet

4 feet

5 feet

4.

11 cm

8 cm

15 cm

5.

7 in.

5 in.

10 in.

6.

11 m

8 m

8 m

7.

7 cm

4 cm

10.5 cm

8.

8.4 cm

3 cm

6.5 cm
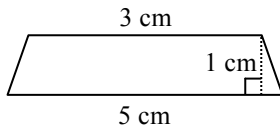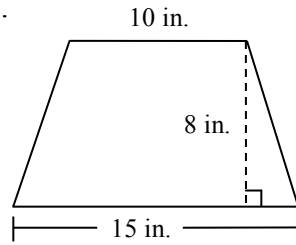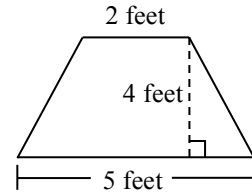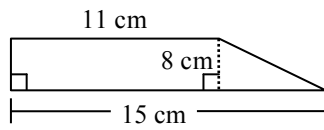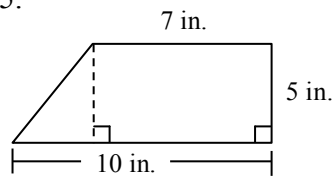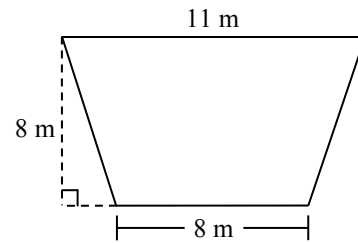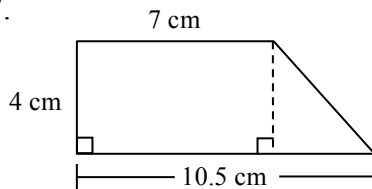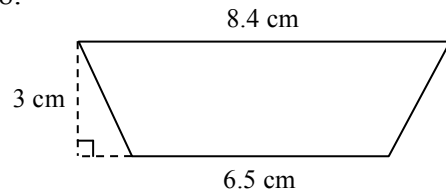
Answers

1.	4 sq. cm	2.	100 sq. in.	3.	14 sq. ft	4.	104 sq. cm
5.	42.5 sq. in.	6.	76 sq. m	7.	35 sq. cm	8.	22.35 sq. cm

   Core Connections, Course 2

AREA OF A TRIANGLE

The area of a triangle is equal to one-half the area of a parallelogram. This fact can easily be shown by cutting a parallelogram in half along a diagonal (see below).

base
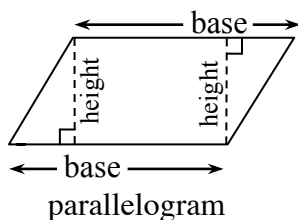
parallelogram

Step 1
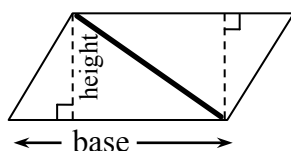
draw a diagonal

Step 2
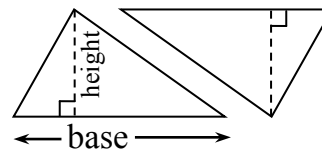
match triangles by cutting apart or by folding

Step 3

As you match the triangles by either cutting the parallelogram apart or by folding along the diagonal, the result is two congruent (same size and shape) triangles. Thus, the area of a triangle has half the area of the parallelogram that can be created from two copies of the triangle.

To find the area of a triangle, follow the steps below.

1. Identify the base.

2. Identify the height.

3. Multiply the base times the height.

4. Divide the product of the base times the height by 2: $A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$

Example 1

Example 2

base = 16 cm

height = 8 cm
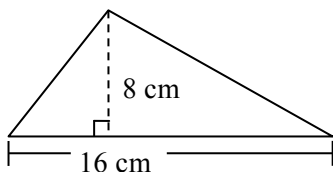
$A = \frac{16 \cdot 8}{2} = \frac{128}{2} = 64 \text{ cm}^2$
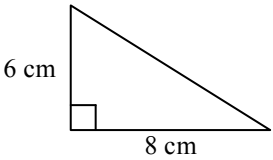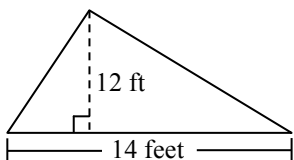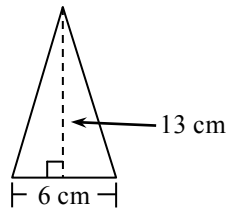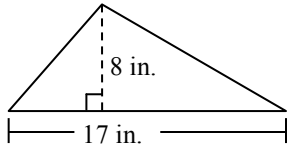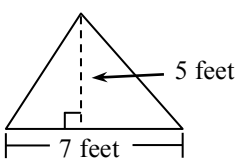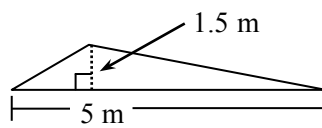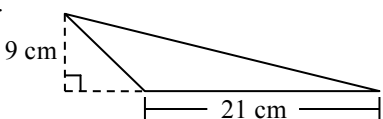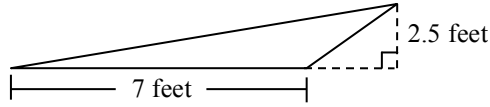
base = 7 cm

height = 4 cm
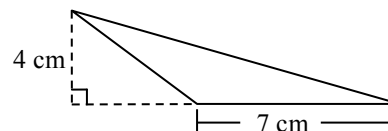
$A = \frac{7 \cdot 4}{2} = \frac{28}{2} = 14 \text{ cm}^2$

Problems

1.

6 cm

8 cm

2.

12 ft

14 feet

3.

13 cm

6 cm

4.

8 in.

17 in.

5.

5 feet

7 feet

6.

1.5 m

5 m

7.

9 cm

21 cm

8.

2.5 feet

7 feet

Answers

1. 24 sq. cm
2. 84 sq. ft
3. 39 sq. cm
4. 68 sq. in.

5. 17.5 sq. ft
6. 3.75 sq. m
7. 94.5 sq. cm
8. 8.75 sq. ft

 Core Connections, Course 2

CALCULATING COMPLEX AREAS USING SUBPROBLEMS

Students can use their knowledge of areas of polygons to find the areas of more complicated figures. The use of subproblems (that is, solving smaller problems in order to solve a larger problem) is one way to find the areas of complicated figures.

Example 1

Find the area of the figure at right.
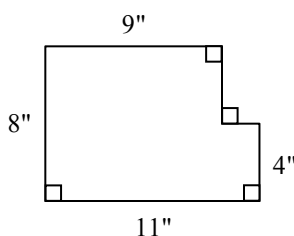
| | Method #1 | Method #2 | Method #3 |
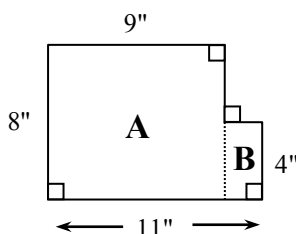
Subproblems:

1. Find the area of rectangle A:

    $8 \cdot 9 = 72$ square inches

2. Find the area of rectangle B:

    $4 \cdot (11 - 9) = 4 \cdot 2 = 8$ square inches

3. Add the area of rectangle A to the area of rectangle B:

    $72 + 8 = 80$ square inches
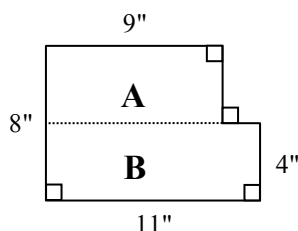
Subproblems:

1. Find the area of rectangle A:

    $9 \cdot (8 - 4) = 9 \cdot 4 = 36$ square inches

2. Find the area of rectangle B:

    $11 \cdot 4 = 44$ square inches

3. Add the area of rectangle A to the area of rectangle B:

    $36 + 44 = 80$ square inches
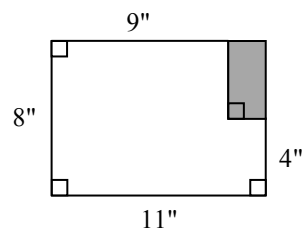
Subproblems:

1. Make a large rectangle by enclosing the upper right corner.

2. Find the area of the new, larger rectangle:

    $8 \cdot 11 = 88$ square inches

3. Find the area of the shaded rectangle:

    $(8 - 4) \cdot (11 - 9)$
    $= 4 \cdot 2 = 8$ square inches

4. Subtract the shaded rectangle from the larger rectangle:

    $88 - 8 = 80$ square inches

Example 2

Find the area of the figure at right.

10 cm   10 cm

6 cm

8 cm
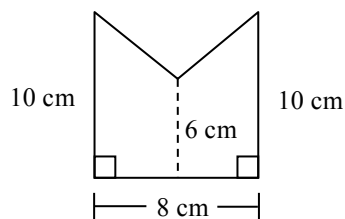
Subproblems:

1. Make a rectangle out of the figure by enclosing the top.
2. Find the area of the entire rectangle: $8 \cdot 10 = 80$ square cm
3. Find the area of the shaded triangle. Use the formula $A = \frac{1}{2}bh$.
   $b = 8$ and $h = 10 - 6 = 4$, so $A = \frac{1}{2}(8 \cdot 4) = \frac{32}{2} = 16$ square cm.
4. Subtract the area of the triangle from the area of the rectangle:
   $80 - 16 = 64$ square cm.
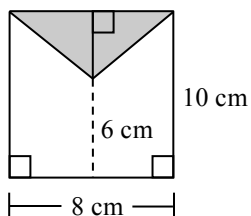
Problems

Find the areas of the figures below.

1.

10'
7'
6'
20'

2.

7 m
18 m
11 m
16 m

3.

15"
19"
9"
17"

4.

6 yards
2 yards
3 yards
10 yards

5.

8 m
10 m
5 m
14 m

6.

15 m
8 m
15 m

7.

7 cm   5 cm
3 cm
12 cm
24 cm

8.

7'
2'
20'
10'
8'
22'

9.

10 cm
7 cm
6 cm
2 cm   4 cm

 Core Connections, Course 2

10.

12 m

8 m

16 m

18 m
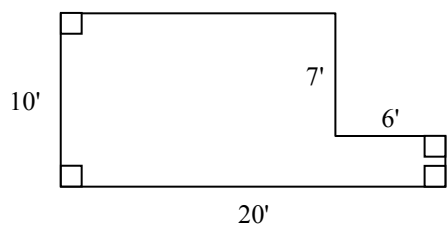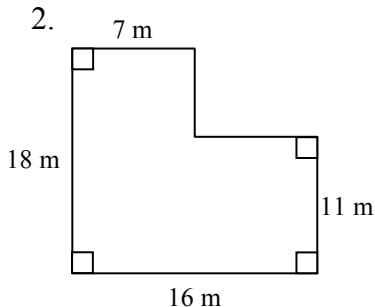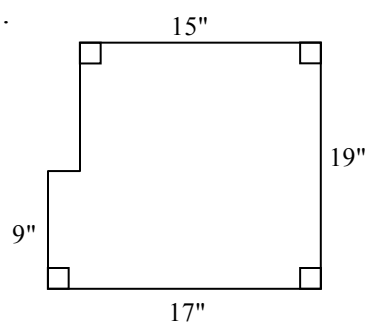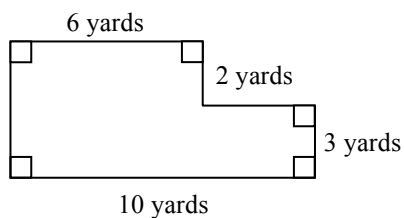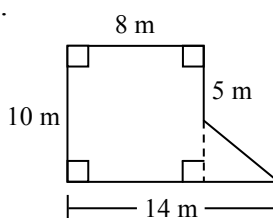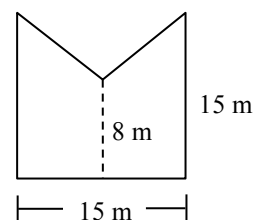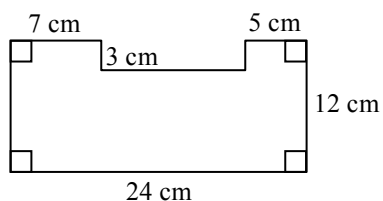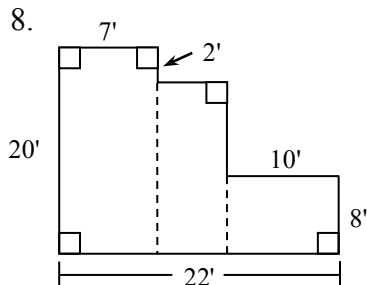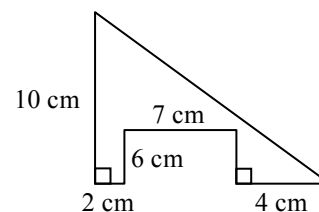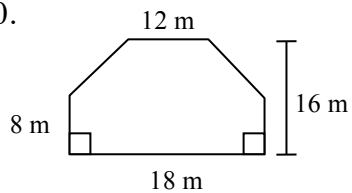
11. Find the area of the shaded region.

9"

12"

7"

15"
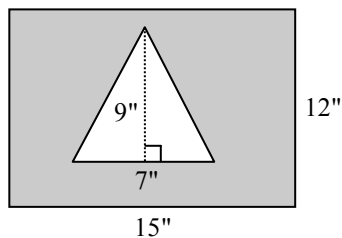
12. Find the area of the shaded region.

14'

12' 8' 7'
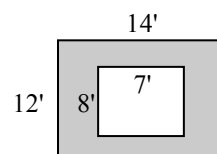
Answers

1. 158 sq. ft

2. 225 sq. m

3. 303 sq. in.

4. 42 sq. yards

5. 95 sq. m

6. 172.5 sq. m

7. 252 sq. cm

8. 310 sq. ft

9. 23 sq. cm

10. 264 sq. m

11. 148.5 sq. in.

12. 112 sq. ft

SURFACE AREA OF A PRISM

The surface area of a prism is the sum of the areas of all of the faces, including the bases. Surface area is expressed in square units.

For additional information, see the Math Notes box in Lesson 9.2.4 of the *Core Connections, Course 2* text.

Example

Find the surface area of the triangular prism at right.

Step 1: Area of the 2 bases: $2\left[\frac{1}{2}(6\text{ cm})(8\text{ cm})\right] = 48\text{ cm}^2$

Step 2: Area of the 3 lateral faces

      Area of face 1: $(6\text{ cm})(7\text{ cm}) = 42\text{ cm}^2$
      Area of face 2: $(8\text{ cm})(7\text{ cm}) = 56\text{ cm}^2$
      Area of face 3: $(10\text{ cm})(7\text{ cm}) = 70\text{ cm}^2$

Step 3: Surface Area of Prism = sum of area of bases and area of the lateral faces:

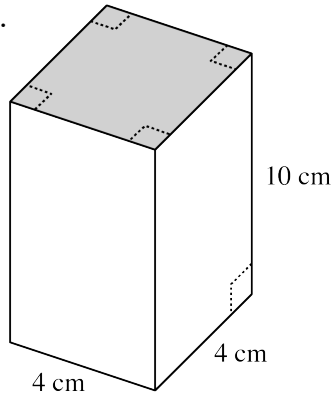 $SA = 48\text{ cm}^2 + 42\text{ cm}^2 + 56\text{ cm}^2 + 70\text{ cm}^2 = 216\text{ cm}^2$
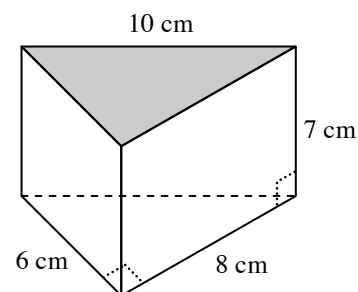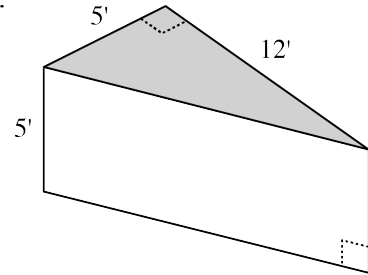
Problems

Find the surface area of each prism.

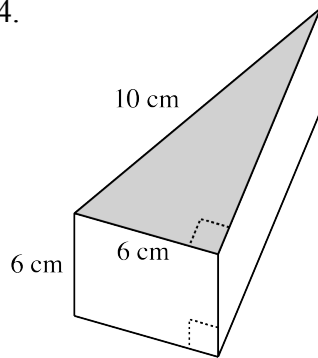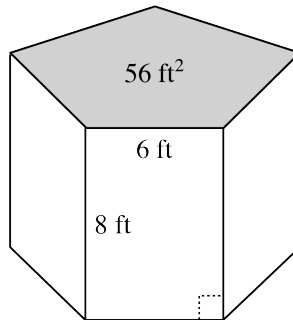1.

5 mm

8 mm

9 mm

2.

10 cm

4 cm

4 cm

3.

5'

12'

5'

4.

10 cm

6 cm

6 cm
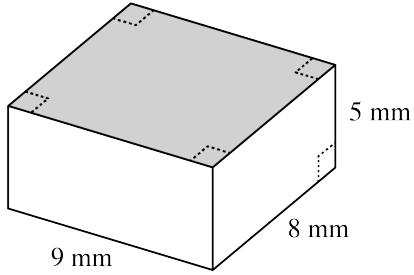
5. The pentagon is equilateral.

56 ft²

6 ft

8 ft

6.

10 cm

2 cm

6 cm

6 cm

10 cm
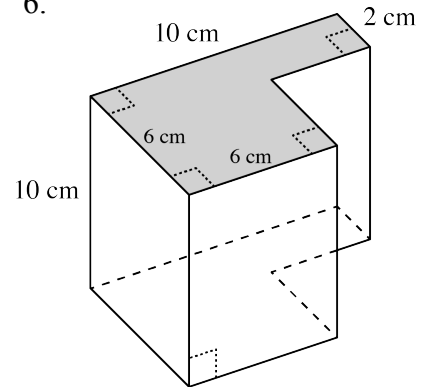
Answers

1.   314 mm^2

2.   192 cm^2

3.   210 ft^2

4.   192 cm^2

5.   344 ft^2

6.   408 cm^2

VOLUME OF A PRISM

Volume is a three-dimensional concept. It measures the amount of interior space of a three-dimensional figure based on a cubic unit, that is, the number of 1 by 1 by 1 cubes that will fit inside a figure.
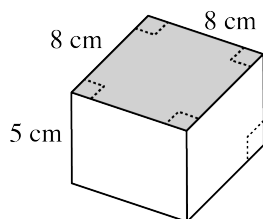
The volume of a prism is the area of either base (B) multiplied by the height (h) of the prism.

$$V = \textbf{(Area of base)} \cdot \textbf{(height)} \text{ or } V = Bh$$

For additional information, see the Math Notes box in Lesson 9.2.4 of the *Core Connections, Course 2* text.
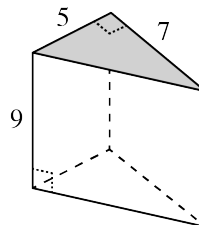
Example 1

Find the volume of the square prism below.

The base is a square with area (B)
$8 \cdot 8 = 64$ units^2.

$$
\begin{aligned}
\text{Volume} &= B(h) \\
&= 64(5) \\
&= 320 \text{ units}^3
\end{aligned}
$$

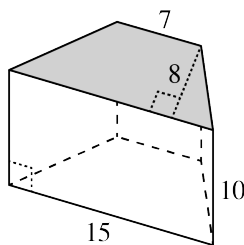## Example 2

Find the volume of the triangular prism below.

The base is a right triangle with area
$\frac{1}{2}(5)(7) = 17.5$ units^2.

$$
\begin{aligned}
\text{Volume} &= B(h) \\
&= 17.5(9) \\
&= 157.5 \text{ units}^3
\end{aligned}
$$

Example 3

Find the volume of the trapezoidal prism below.

The base is a trapezoid with
area $\frac{1}{2}(7+15) \cdot 8 = 88$ units^2.

$$
\begin{aligned}
\text{Volume} \\
&= B(h) \\
&= 88(10) \\
&= 880 \text{ units}^3
\end{aligned}
$$

Example 4

Find the height of a right prism with a volume of 132.5 cm^3 and base area of 25 cm^2.

$$
\begin{aligned}
\text{Volume} &= B(h) \\
132.5 &= 25(h) \\
h &= \frac{132.5}{25} \\
h &= 5.3 \text{ cm}
\end{aligned}
$$

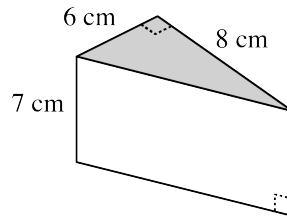 *Core Connections, Course 2*

Problems

Calculate the volume of each prism. The base of each figure is shaded.

1.   Rectangular Prism

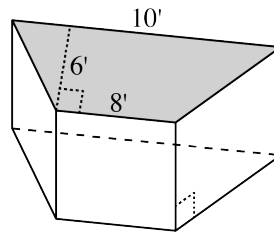4 feet

3 feet

1 foot

2.   Right Triangular Prism

6 cm

8 cm

7 cm

3.   Rectangular Prism

5 in.   6 in.

8.5 in.
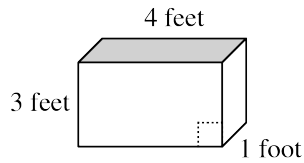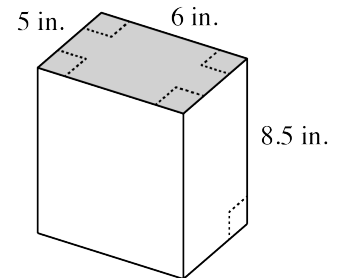
4.   Right Triangular Prism

7.2 cm

4.5 cm

4 cm
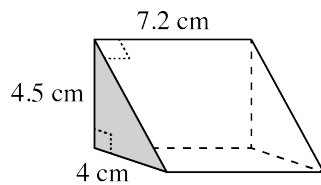
5.   Trapezoidal Prism

10'

6'

8'

6.   Triangular Prism with $B = 15\frac{1}{2}$ cm^2

$8\frac{1}{4}$ cm

$15\frac{1}{2}$ cm^2
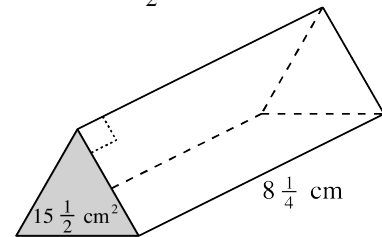
7.   Find the volume of a right prism with base area 32 cm^2 and height 1.5 cm.

8.   Find the height of a right prism with base area 32 cm^2 and volume 176 cm^3.

9.   Find the base area of a right prism with volume 47.01 cm^3 and height 3.2 cm.

Answers

1.   12 ft^3 2.   168 cm^3 3.   240 in.^{3} 4.   64.8 cm^3 5.   324 ft^3

6. $127\frac{7}{8}$ cm^3 7.   48 cm^3 8.   5.5 cm   9.   14.7 cm^2