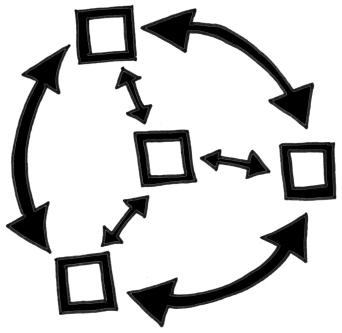
Core Connections, Course 3
Parent Guide with Extra Practice



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Introduction to the Parent Guide with Extra Practice

Welcome to the *Core Connections, Course 3 Parent Guide with Extra Practice*. The purpose of this guide is to assist you should your child need help with homework or the ideas in the course. We believe all students can be successful in mathematics as long as they are willing to work and ask for help when they need it. We encourage you to contact your child's teacher if your student has additional questions that this guide does not answer.

Detailed examples follow a summary of the concept or skill and include complete solutions. The examples are similar to the work your child has done in class. Additional problems, with answers, are provided for your child to try.

There will be some topics that your child understands quickly and some concepts that may take longer to master. The big ideas of the course take time to learn. This means that students are not necessarily expected to master a concept when it is first introduced. When a topic is first introduced in the textbook, there will be several problems to do for practice. Succeeding lessons and homework assignments will continue to practice the concept or skill over weeks and months so that mastery will develop over time.

Practice and discussion are required to understand mathematics. When your child comes to you with a question about a homework problem, often you may simply need to ask your child to read the problem and then ask what the problem is asking. Reading the problem aloud is often more effective than reading it silently. When you are working problems together, have your child talk about the problems. Then have your child practice on his/her own.

Below is a list of additional questions to use when working with your child. These questions do not refer to any particular concept or topic. Some questions may or may not be appropriate for some problems.

- What have you tried? What steps did you take?
- What didn't work? Why didn't it work?
- What have you been doing in class or during this chapter that might be related to this problem?
- What does this word/phrase tell you?
- What do you know about this part of the problem?
- Explain what you know right now.
- What do you need to know to solve the problem?
- How did the members of your study team explain this problem in class?
- What important examples or ideas were highlighted by your teacher?
- Can you draw a diagram or sketch to help you?
- Which words are most important? Why?
- What is your guess/estimate/prediction?
- Is there a simpler, similar problem we can do first?
- How did you organize your information? Do you have a record of your work?
- Have you tried drawing a diagram, making a list, looking for a pattern, etc.?

If your student has made a start at the problem, try these questions.

- What do you think comes next? Why?
- What is still left to be done?
- Is that the only possible answer?
- Is that answer reasonable?
- How could you check your work and your answer?
- How could your method work for other problems?

If you do not seem to be making any progress, you might try these questions.

- Let's look at your notebook, class notes, and Toolkit. Do you have them?
- Were you listening to your team members and teacher in class? What did they say?
- Did you use the class time working on the assignment? Show me what you did.
- Were the other members of your team having difficulty with this as well? Can you call your study partner or someone from your study team?

This is certainly not a complete list; you will probably come up with some of your own questions as you work through the problems with your child. Ask any question at all, even if it seems too simple to you.

To be successful in mathematics, students need to develop the ability to reason mathematically. To do so, students need to think about what they already know and then connect this knowledge to the new ideas they are learning. Many students are not used to the idea that what they learned yesterday or last week will be connected to today's lesson. Too often students do not have to do much thinking in school because they are usually just told what to do. When students understand that connecting prior learning to new ideas is a normal part of their education, they will be more successful in this mathematics course (and any other course, for that matter). The student's responsibilities for learning mathematics include the following:

- Actively contributing in whole class and study team and discussions.
- Completing (or at least attempting) all assigned problems and turning in assignments in a timely manner.
- Checking and correcting problems on assignments (usually with their study partner or study team), based on answers and solutions provided in class and online.
- Asking for help when needed from his or her study partner, study team, and/or teacher.
- Attempting to provide help when asked by other students.
- Taking notes and using his/her Toolkit when recommended by the teacher or the text.
- Keeping a well-organized notebook.
- Not distracting other students from the opportunity to learn.

Assisting your child to understand and accept these responsibilities will help him or her to be successful in this course, develop mathematical reasoning, and form habits that will help her/him become a life-long learner.

Additional support for students and parents is provided at the CPM website (cpm.org) and at the CPM Homework Help website (homework.cpm.org).

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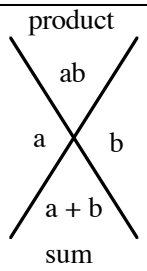
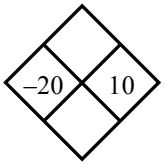
DIAMOND PROBLEMS 1.1.1

In every Diamond Problem, the product of the two side numbers (left and right) is the top number and their sum is the bottom number.

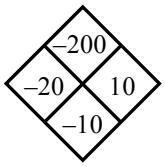
Diamond Problems are an excellent way of practicing addition, subtraction, multiplication, and division of positive and negative integers, decimals and fractions. They have the added benefit of preparing students for factoring binomials in algebra.



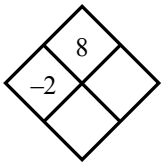
Example 1



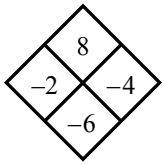
The top number is the product of –20 and 10, or –200. The bottom number is the sum of –20 and 10, or –20 + 10 = –10.



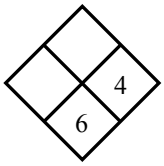
Example 2



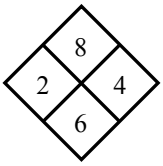
The product of the right number and –2 is 8. Thus, if you divide 8 by – 2 you get – 4, the right number.
The sum of –2 and – 4 is – 6, the bottom number.



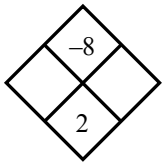
Example 3



To get the left number, subtract 4 from 6,
6 – 4 = 2. The product of 2 and 4 is 8, the top number.



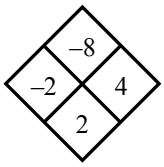
Example 4



The easiest way to find the side numbers in a situation like this one is to look at all the pairs of factors of –8. They are:

–1 and 8, –2 and 4, –4 and 2, and –8 and 1.

Only one of these pairs has a sum of 2: –2 and 4. Thus, the side numbers are –2 and 4.



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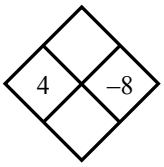
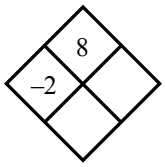
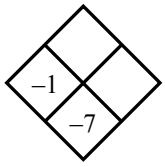
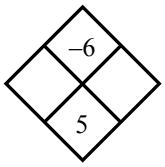
Problems

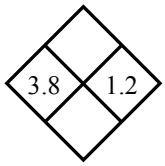
Complete each of the following Diamond Problems.

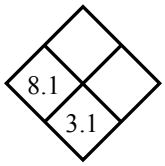
1. 

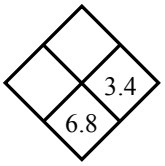
2. 

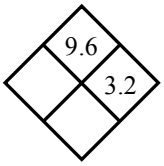
3. 

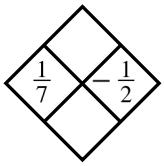
4. 

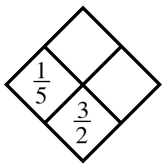
5. 

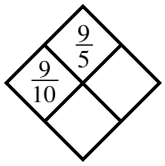
6. 

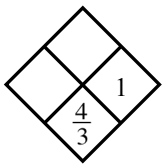
7. 

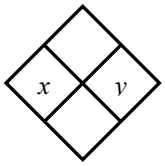
8. 

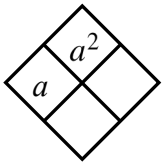
9. 

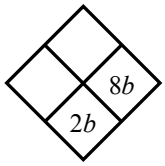
10. 

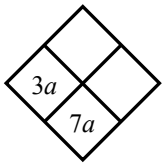
11. 

12. 

13. 

14. 

15. 

16. 

Answers

1. –32 and –4
2. –4 and –6
3. –6 and 6
4. 6 and –1
5. 4.56 and 5
6. –5 and –40.5
7. 3.4 and 11.56
8. 3 and 6.2
9. $-\frac{1}{14}$ and $-\frac{5}{14}$
10. $\frac{13}{10}$ and $\frac{13}{50}$
11. 2 and $\frac{29}{10}$
12. $\frac{1}{3}$ and $\frac{1}{3}$
13. xy and $x + y$
14. a and $2a$
15. $-6b$ and $-48b^2$
16. $4a$ and $12a^2$

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DESCRIBING AND EXTENDING PATTERNS 1.1.2

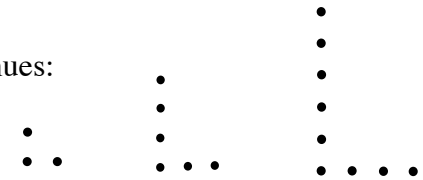
Students are asked to use their observations and pattern recognition skills to extend patterns and predict the number of dots that will be in a figure that is too large to draw. Later, variables will be used to describe the patterns.

Example

Examine the dot pattern at right. Assuming the pattern continues:

a. Draw Figure 4.

b. How many dots will be in Figure 10?



Solution:

The horizontal dots are one more than the figure number and the vertical dots are even numbers (or, twice the figure number).

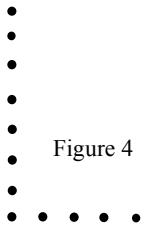


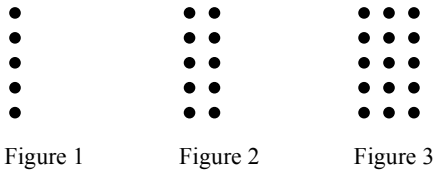
Figure 1 has 3 dots, Figure 2 has 6 dots, and Figure 3 has 9 dots. The number of dots is the figure number multiplied by three.

Figure 10 has 30 dots.

Problems

For each dot pattern, draw the next figure and determine the number of dots in Figure 10.

1.



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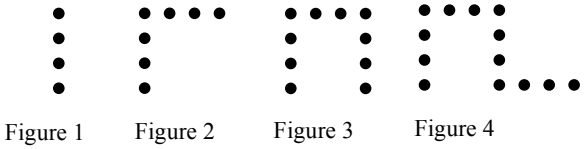

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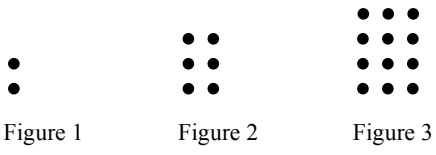

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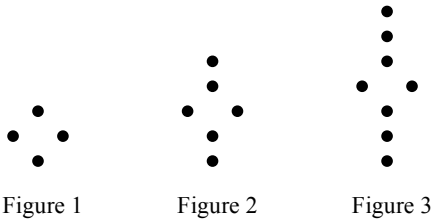

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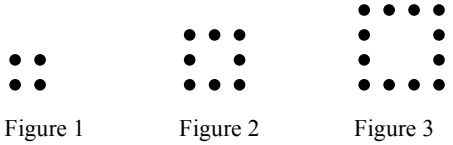
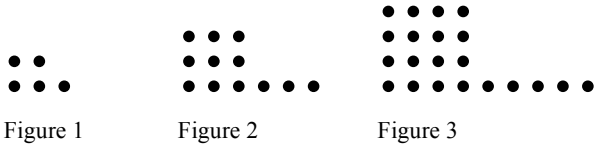


6.



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Answers

1. 50 dots

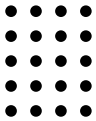

Figure 4

2. 31 dots

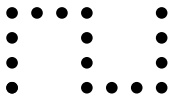

Figure 5

3. 110 dots

Figure 4

4. 22 dots

Figure 4

5. 40 dots

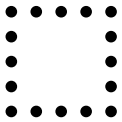

Figure 5

6. 140 dots

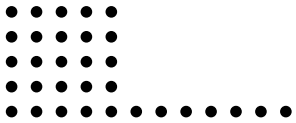

Figure 4

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FOUR-QUADRANT GRAPHING 1.1.2 and 1.1.3

The graphing that was started in earlier grades is now extended to include negative values, and students will graph algebraic equations with two variables.

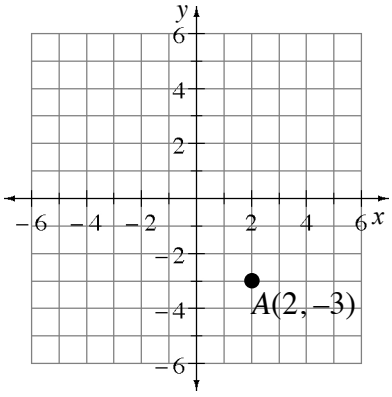
GRAPHING POINTS

Points on a coordinate graph are identified by two numbers in an ordered pair written as (x, y). The first number is the x–coordinate of the point and the second number is the y–coordinate. Taken together, the two coordinates name exactly one point on the graph. The examples below show how to place a point on an xy–coordinate graph.

Example 1

Graph point $A(2, -3)$.

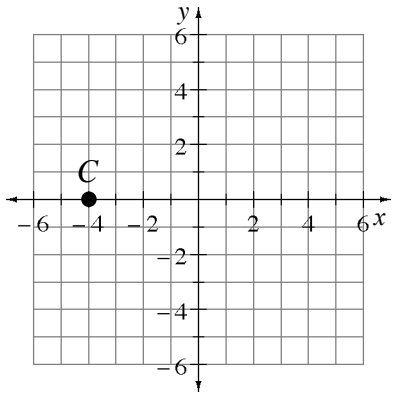
Go right 2 units from the origin $(0, 0)$, then go down 3 units. Mark the point.



Example 2

Plot the point $C(-4, 0)$ on a coordinate grid.

Go to the left from the origin 4 units, but do not go up or down. Mark the point.



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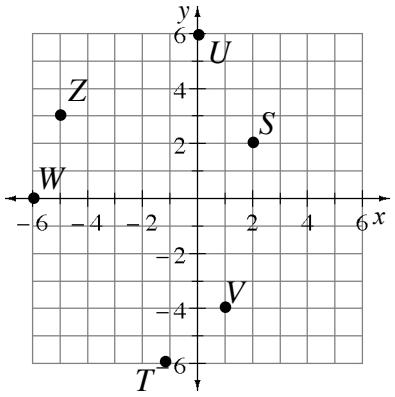
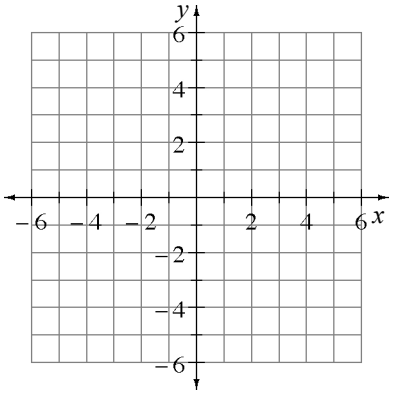
Problems

1. Name the coordinate pair for each point shown on the grid below.



2. Use the ordered pair to locate each point on a coordinate grid. Place a "dot" at each point and label it with its letter name.

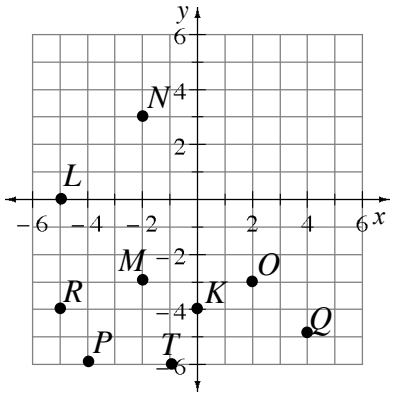
$K(0, -4)$
$L(-5, 0)$
$M(-2, -3)$
$N(-2, 3)$
$O(2, -3)$
$P(-4, -6)$
$Q(4, -5)$
$R(-5, -4)$
$T(-1, -6)$



Answers

1. $S(2, 2)$
   $T(-1, -6)$
   $U(0, 6)$
   $V(1, -4)$
   $W(-6, 0)$
   $Z(-5, 3)$

2.



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WRITING EQUATIONS FOR WORD PROBLEMS (THE 5-D PROCESS)

1.1.3 Math Notes

At first students used the 5-D Process to solve problems. However, solving complicated problems with the 5-D Process can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the 5-D Process can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to use the 5-D Process. Most of the problems here will not be complex so that you can practice writing equations using the 5-D Process. The same example problems previously used are used here except they are now extended to writing and solving equations.

Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

Describe: Number of nectarines is three times the number of grapefruit.
Number of nectarines plus number of grapefruit equals 36.

	Define		**Do**	**Decide**
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high
Trial 2:	10	30	40	too high

After several trials to establish a pattern in the problem, you can generalize it using a variable. Since we could try any number of grapefruit, use x to represent it. The pattern for the number of oranges is three times the number of grapefruit, or $3x$. The total pieces of fruit is the sum of column one and column two, so our table becomes:

Define		**Do**	**Decide**
# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
x	$3x$	$x + 3x$	$= 36$

Since we want the total to agree with the check, our equation is $x + 3x = 36$. Simplifying this yields $4x = 36$, so $x = 9$ (grapefruit) and then $3x = 27$ (nectarines).

Declare: There are 9 grapefruit and 27 nectarines.

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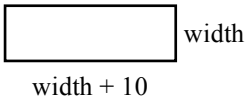
Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is 10 feet more than the width, what are the dimensions (length and width) of the rectangle?

Describe/Draw:



	Define		**Do**	**Decide**
	Width	Length	Perimeter	120?
Trial 1:	10	25	$(10 + 25) \cdot 2 = 70$	too low
Trial 2:	20	30	100	too low

Again, since we could guess any width, we labeled this column x. The pattern for the second column is that it is 10 more than the first: $x + 10$. The perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

Define		**Do**	**Decide**
Width	Length	Perimeter	120?
x	$x + 10$	$(x + x + 10) \cdot 2$	$= 120$

Solving the equation:

$$(x + x + 10) \cdot 2 = 120$$
$$2x + 2x + 20 = 120$$
$$4x + 20 = 120$$
$$4x = 100$$

So $x = 25$ (width) and $x + 10 = 35$ (length).

Declare: The width is 25 feet and the length is 35 feet.

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Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth $2.40. How many dimes and quarters does Jorge have?

Describe: The number of quarters plus 10 equals the number of dimes.
The total value of the coins is $2.40.

	Define				**Do**	**Decide**
	Quarters	Dimes	Value of Quarters	Value of Dimes	Total Value	$2.40?
Trial 1:	10	20	2.50	2.00	4.50	too high
Trial 2:	8	18	2.00	1.80	3.80	too high
	x	$x + 10$	$0.25x$	$0.10(x + 10)$	$0.25x + 0.10(x + 10)$	

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes x, but then in the table the Value of Quarters column is $0.25x$. Thus the number of dimes is $x + 10$, but the value of dimes is $0.10(x + 10)$. Finally, to find the numbers, the equation becomes $0.25x + 0.10(x + 10) = 2.40$.

Solving the equation:
$$\begin{aligned} 0.25x + 0.10x + 1.00 &= 2.40 \\ 0.35x + 1.00 &= 2.40 \\ 0.35x &= 1.40 \\ x &= 4.00 \end{aligned}$$

Declare: There are 4 quarters worth $1.00 and 14 dimes worth $1.40 for a total value of $2.40.

Problems

Start the problems using the 5-D Process. Then write an equation. Solve the equation.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?

3. Tomás is thinking of a number. If he triples his number and subtracts 13, the result is 305. Of what number is Tomás thinking?

4. Two consecutive numbers have a sum of 123. What are the two numbers?

5. Two consecutive even numbers have a sum of 246. What are the numbers?

6. Joe's age is three times Aaron's age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina's age? Joe's age? Aaron's age?

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7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?

8. A wood board 156 centimeters long is cut into three parts. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?

9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate column titles for, "Number of Nickels," "Value of Nickels," "Number of Dimes," and "Value of Dimes.")

10. Tickets to the school play are $ 5.00 for adults and $ 3.50 for students. If the total value of all the tickets sold was $2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?

11. A wood board 250 centimeters long is cut into five pieces: three short ones of equal length and two that are both 15 centimeters longer than the shorter ones. What are the lengths of the boards?

12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth $ 9.60, how many nickels, dimes, and quarters are there?

Answers (Equations may vary.)

1. $x + (x + 26) = 100$
The lengths of the boards are 37 cm and 63 cm.

2. $x + (x + 5) = 51$
Thu is 28 years old and her brother is 23 years old.

3. $3x - 13 = 305$
Tomás is thinking of the number 106.

4. $x + (x + 1) = 123$
The two consecutive numbers are 61 and 62.

5. $x + (x + 2) = 246$
The two consecutive even numbers are 122 and 124.

6. $x + (x + 6) + 3(x + 6) = 149$
Christine is 25, Aaron is 31, and Joe is 93 years old.

7. $2x + 4(38 - x) = 116$
Farmer Fran has 20 goats and 18 chickens.

8. $x + (x + 15) + (x + 15) = 156$
The lengths of the boards are 42, 57, and 57 cm.

9. $0.05x + 0.10(15 - x) = 0.90$
Juan has 12 nickels and 3 dimes.

10. $\$5x + \$3.50(x + 100) = 2517.50$
There were 255 adult and 355 student tickets purchased for the play.

11. $3x + 2(x + 15) = 250$
The lengths of the boards are 44 and 59 cm.

12. $0.05x + 0.25x + 0.10(3x) = 9.60$
Conrad has 16 quarters, 16 nickels, and 48 dimes.

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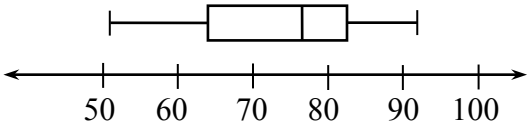
BOX PLOTS

One way to display a distribution of one-variable numerical data is with a box plot. A box plot is the only display of data that clearly shows the median, quartiles, range, and outliers of a data set.

Example 1

Display this data in a box plot:
51, 55, 55, 62, 65, 72, 76, 78, 79, 82, 83, 85, 91, and 93.

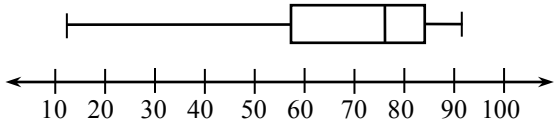
- Since this data is already in order from least to greatest, the range is 93 – 51 = 42. Thus you start with a number line with equal intervals from 50 to 100.
- The median of the set of data is 77. A vertical segment is drawn at this value above the number line.
- The median of the lower half of the data (the first quartile) is 62. A vertical segment is drawn at this value above the number line.
- The median of the upper half of the data (the third quartile) is 83. A vertical segment is drawn at this value above the number line.
- A box is drawn between the first and third quartiles.
- Place a vertical segment at the minimum value (51) and at the maximum value (93). Use a line segment to connect the minimum to the box and the maximum to the box.



Example 2

Display this data in a box plot:
62, 65, 93, 51, 12, 79, 85, 55, 72, 78, 83, 91, and 76.

- Place the data in order from least to greatest: 12, 51, 55, 62, 65, 72, 76, 78, 79, 83, 85, 91, 93. The range is 93 – 12 = 81. Thus you want a number line with equal intervals from 10 to 100.
- Find the median of the set of data: 76. Draw the line segment.
- Find the first quartile: 55 + 62 = 117; 117 ÷ 2 = 58.5. Draw the line segment.
- Find the third quartile: 83 + 85 = 168; 168 ÷ 2 = 84. Draw the line segment.
- Draw the box connecting the first and third quartiles. Place a line segment at the minimum value (12) and a line segment at the maximum value (93). Connect the minimum and maximum values to the box.



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Problems

Create a box plot for each set of data.

1. 45, 47, 52, 85, 46, 32, 83, 80, and 75.

2. 75, 62, 56, 80, 72, 55, 54, and 80.

3. 49, 54, 52, 58, 61, 72, 73, 78, 73, 82, 83, 73, 61, 67, and 68.

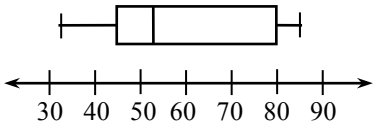
4. 65, 35, 48, 29, 57, 87, 94, 68, 86, 73, 58, 74, 85, 91, 88, and 97.

5. 265, 263, 269, 259, 267, 264, 253, 275, 264, 260, 273, 257, and 291.

6. 48, 42, 37, 29, 49, 46, 38, 28, 45, 45, 35, 46.25, 34, 46, 46.5, 43, 46.5, 48, 41.25, 29, and 47.75.

Answers

1.



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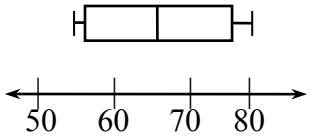

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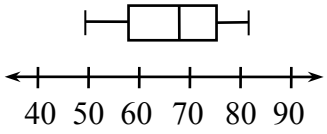

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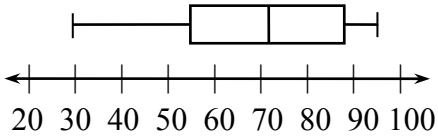

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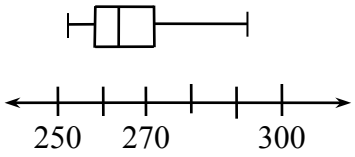

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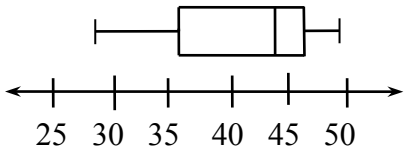

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PROPORTIONAL RELATIONSHIPS 1.2.1 and 1.2.2

A **proportion** is an equation stating the two ratios (fractions) are equal. Two values are in a proportional relationship if a proportion may be set up to relate the values.

For more information, see the Math Notes boxes in Lessons 1.2.2 and 7.2.5 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 3* Checkpoint 3 materials.

Example 1

The average cost of a pair of designer jeans has increased $15 in 4 years. What is the unit growth rate (dollars per year)?

Solution: The growth rate is $\frac{15 \text{ dollars}}{4 \text{ years}}$. To create a unit rate we need a denominator of "one." $\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{x \text{ dollars}}{1 \text{ year}}$. Using a Giant One: $\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{4}{4} \cdot \frac{x \text{ dollars}}{1 \text{ year}} \Rightarrow 3.75 \frac{\text{dollars}}{\text{year}}$.

Example 2

Ryan's famous chili recipe uses 3 tablespoons of chili powder for 5 servings. How many tablespoons are needed for the family reunion needing 40 servings?

Solution: The rate is $\frac{3 \text{ tablespoons}}{5 \text{ servings}}$ so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

$$\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3}{5} \cdot \frac{8}{8} = \frac{24}{40} \Rightarrow t = 24$$

Another method is to use cross multiplication:

$$\frac{3}{5} = \frac{t}{40}$$

$$\frac{3}{5} \times \frac{t}{40}$$

$$5 \cdot t = 3 \cdot 40$$

$$5t = 120$$

$$t = 24$$

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5}s$ represents the general proportional situation and one could substitute the number of servings needed into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method the answer is 24 tablespoons.

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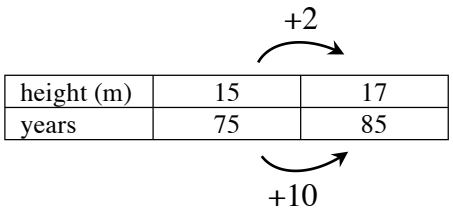
Example 3

Based on the table at right, what is the unit growth rate (meters per year)?



Solution: $\frac{2 \text{ meters}}{10 \text{ years}} = \frac{2 \text{ meters} \cdot \frac{1}{10}}{10 \text{ years} \cdot \frac{1}{10}} = \frac{\frac{1}{5} \text{ meter}}{1 \text{ year}} = \frac{1}{5} \frac{\text{meter}}{\text{year}}$

Problems

For problems 1 through 10 find the unit rate. For problems 11 through 25, solve each problem.

1. Typing 731 words in 17 minutes (words per minute)

2. Reading 258 pages in 86 minutes (pages per minute)

3. Buying 15 boxes of cereal for \$43.35 (cost per box)

4. Scoring 98 points in a 40 minute game (points per minute)

5. Buying $2\frac{1}{4}$ pounds of bananas cost \$1.89 (cost per pound)

6. Buying $\frac{2}{3}$ pounds of peanuts for \$2.25 (cost per pound)

7. Mowing $1\frac{1}{2}$ acres of lawn in $\frac{3}{4}$ of a hour (acres per hour)

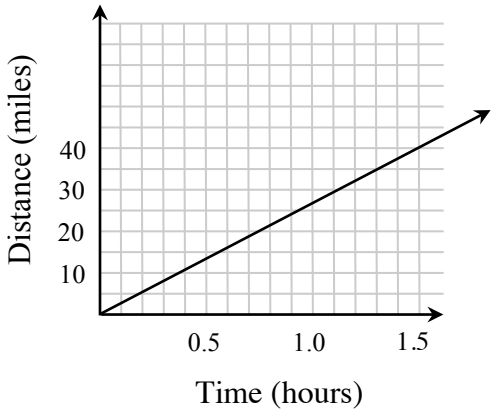
8. Paying \$3.89 for 1.7 pounds of chicken (cost per pound)

9.

weight (g)	6	8	12	20
length (cm)	15	20	30	50

What is the weight per cm?

10. For the graph at right, what is the rate in miles per hour?



11. If a box of 100 pencils costs \$4.75, what should you expect to pay for 225 pencils?

12. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?

13. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?

14. The tax on a \$600 vase is \$54. What should be the tax on a \$1700 vase?

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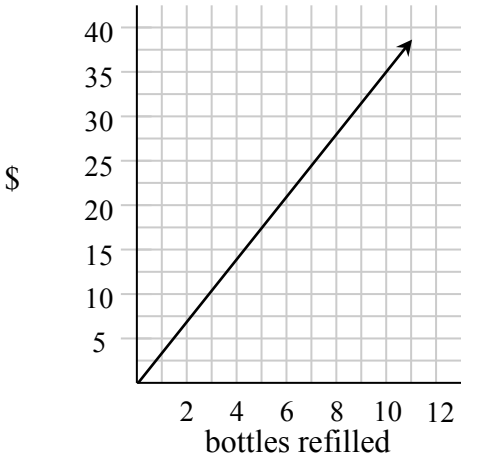
15. Use the table at right to determine how long it will take the Spirit club to wax 60 cars.

cars waxed	8	16	32
hours	3	6	12

16. While baking, Evan discovered a recipe that required $\frac{1}{2}$ cups of walnuts for every $2\frac{1}{4}$ cups of flour. How many cups of walnuts will he need for 4 cups of flour?

17. Based on the graph, what would the cost to refill 50 bottles?



18. Sam grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. How much should he grow in one year?

19. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 60 minutes?

20. If Caitlin needs $1\frac{1}{3}$ cans of paint for each room in her house, how many cans of paint will she need to paint the 7-room house?

21. Stephen receives 20 minutes of video game time every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to work?

22. Sarah's grape vine grew 15 inches in 6 weeks, write an equation to represent its growth after t weeks.

23. On average Max makes 45 out of 60 shots with the basketball, write an equation to represent the average number of shots made out of x attempts.

24. Write an equation to represent the situation in problem 14.

25. Write an equation to represent the situation in problem 17.

Answers

1. $43\ \frac{\text{words}}{\text{minute}}$
2. $3\ \frac{\text{pages}}{\text{minute}}$
3. $2.89\ \frac{\$}{\text{box}}$
4. $2.45\ \frac{\text{points}}{\text{minute}}$
5. $0.84\ \frac{\$}{\text{pound}}$
6. $3.38\ \frac{\$}{\text{pound}}$
7. $2\ \frac{\text{acres}}{\text{hour}}$
8. $2.29\ \frac{\$}{\text{pound}}$
9. $\frac{2}{5}\ \frac{\text{grams}}{\text{centimeter}}$
10. $\approx 27\ \frac{\text{miles}}{\text{hour}}$
11. \$10.69
12. 24.5 min.
13. 19.2 hours
14. \$153
15. 22.5 hours
16. $\frac{8}{9}$ cup
17. \$175
18. $4\frac{2}{3}$ inches
19. ≈ 5.36 miles
20. $9\frac{1}{3}$ cans
21. $3\frac{3}{8}$ hours
22. $g = \frac{5}{2}t$
23. $s = \frac{3}{4}x$
24. $t = 0.09c$
25. $C = 3.5b$

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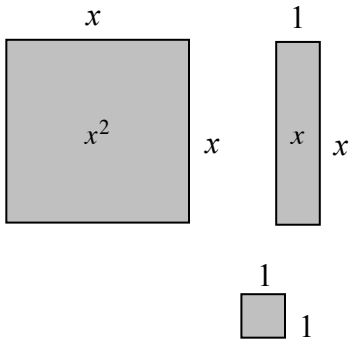
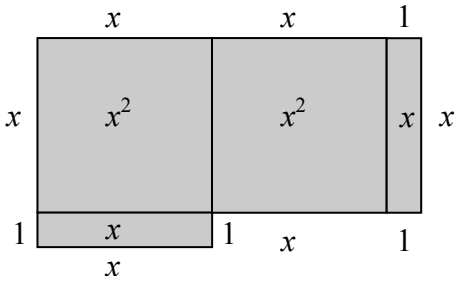
ALGEBRA TILES AND PERIMETER 2.1.1 and 2.1.2

Algebraic expressions can be represented by the perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the tile is named by its area as shown on the tile itself in the figures at right. When using the tiles, perimeter is the distance around the exterior of a figure.

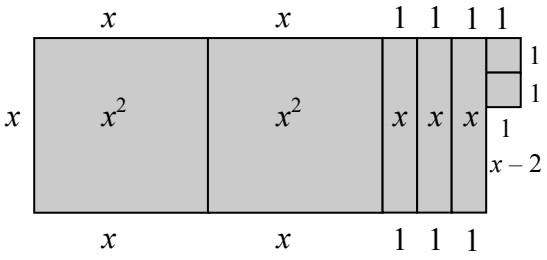


Example 1



P = $6x + 4$ units

Example 2



P = $6x + 8$ units

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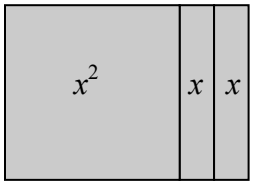
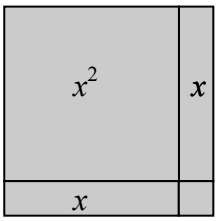
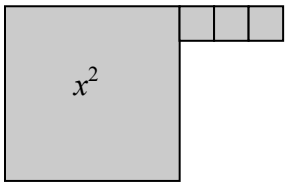
Problems

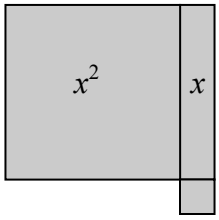
Determine the perimeter of each figure.

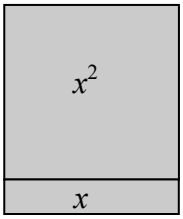
1. 

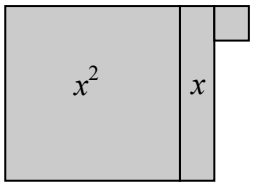
2. 

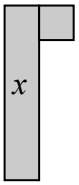
3.

4. 

5. 

6. 

7. 

8. 

Answers

1. $4x + 4$ un.
2. $4x + 4$ un.
3. $2x + 8$ un.
4. $4x + 6$ un.
5. $4x + 4$ un.
6. $4x + 2$ un.
7. $4x + 4$ un.
8. $2x + 4$ un.

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COMBINING LIKE TERMS 2.1.2 and 2.1.3

Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) raised to the same powers, into one term. The skill of combining like terms is necessary for solving equations. For additional information, see the Math Notes box in Lesson 2.1.3 of the *Core Connections, Course 3* text.

Example 1

Combine like terms to simplify the expression $3x + 5x + 7x$.

All these terms have x as the variable, so they are combined into one term, $15x$.

Example 2

Simplify the expression $3x + 12 + 7x + 5$.

The terms with x can be combined. The terms without variables (the constants) can also be combined.

$3x + 12 + 7x + 5$

$3x + 7x + 12 + 5$

$10x + 17$

Note that in the simplified form the term with the variable is listed before the constant term.

Example 3

Simplify the expression $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$.

$5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$

$4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1$

$6x^2 + 8x + 3$

Note that terms with the same variable but with different exponents are not combined and are listed in order of decreasing power of the variable, in simplified form, with the constant term last.

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Example 4

The algebra tiles, as shown in the *Algebra Tiles and Perimeter* section, are used to model how to combine like terms.

The large square represents x^2, the rectangle represents x, and the small square represents one. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small squares. If we want to combine:
$2x^2 + 3x + 4$ and $3x^2 + 5x + 7$, visualize the tiles to help combine the like terms:

$2x^2$ (2 large squares) + $3x$ (3 rectangles) + 4 (4 small squares)
+ $3x^2$ (3 large squares) + $5x$ (5 rectangles) + 7 (7 small squares)

The combination of the two sets of tiles, written algebraically, is: $5x^2 + 8x + 11$.

Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite *like* terms in vertical columns before you combine them:

$$(2x^2 - 5x + 6) + (3x^2 + 4x - 9)$$

$$\textcircled{2x^2} - \textcircled{5x} + \textcircled{6} + \textcircled{3x^2} + \textcircled{4x} - \textcircled{9}$$

$$\begin{array}{r} 2x^2 \quad -5x \quad +6 \\ +\,3x^2 \quad +4x \quad -9 \\ \hline 5x^2 \quad -\ x \quad -3 \end{array}$$

This procedure may make it easier to identify the terms as well as the sign of each term.

Problems

Combine the following sets of terms.

1. $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$
2. $(3x^2 + x + 4) + (x^2 + 4x + 7)$
3. $(8x^2 + 3) + (4x^2 + 5x + 4)$
4. $(4x^2 + 6x + 5) - (3x^2 + 2x + 4)$
5. $(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$
6. $(3x^2 - 7x) - (x^2 + 3x - 9)$
7. $(5x + 6) + (-5x^2 + 6x - 2)$
8. $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$
9. $3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x$
10. $2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2$

Answers

1. $6x^2 + 8x + 13$
2. $4x^2 + 5x + 11$
3. $12x^2 + 5x + 7$
4. $x^2 + 4x + 1$
5. $6x^2 - 9x - 2$
6. $2x^2 - 10x + 9$
7. $-5x^2 + 11x + 4$
8. $7x + 2$
9. $-c^2 + 4c + x - 3$
10. $3a^3 - 2a^2 + 2a + 14$

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SIMPLIFYING EXPRESSIONS (ON AN EXPRESSION MAT) 2.1.3 – 2.1.5

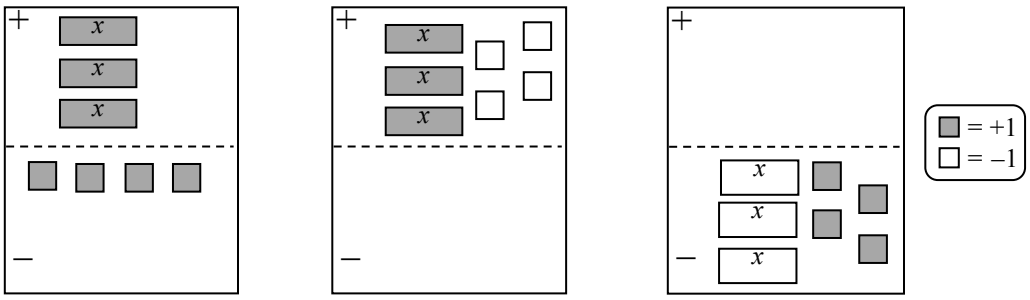
Two Region Expression Mats

An Expression Mat is an organizational tool that is used to represent algebraic expressions. Pairs of Expression Mats can be modified to make an Equation Mat. The upper half of an Expression Mat is the positive region and the lower half is the negative region. Positive algebra tiles are shaded and negative tiles are blank. A matching pair of tiles with one tile shaded and the other one blank represents zero (0).

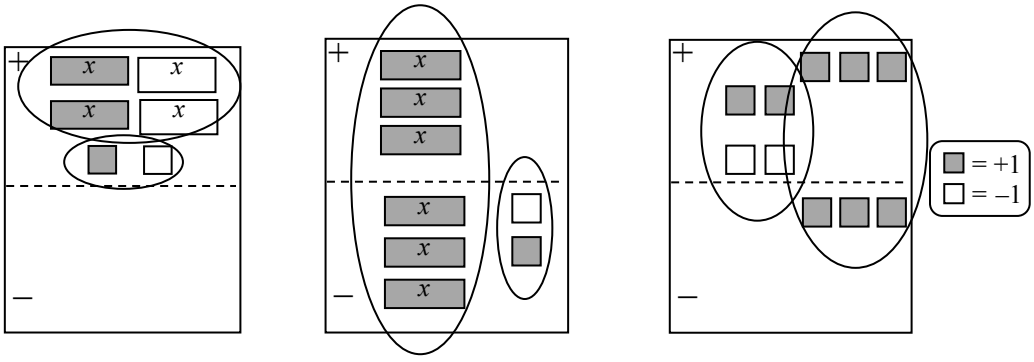
Tiles may be removed from or moved on an expression mat in one of three ways: (1) removing the same number of opposite tiles in the same region; (2) flipping a tile from one region to another. Such moves create "opposites" of the original tile, so a shaded tile becomes un-shaded and an un-shaded tile becomes shaded; and (3) removing an equal number of identical tiles from both the "+" and "–" regions. See the Math Notes box in Lesson 2.1.6 of the *Core Connections, Course 3* text.

Examples

$3x - 4$ can be represented various ways.



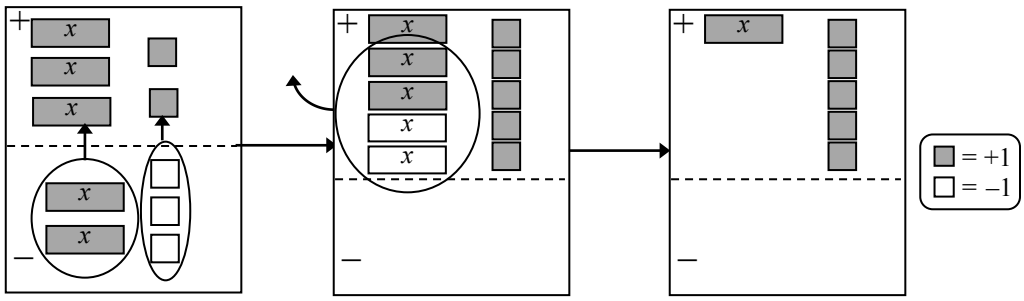
The Expression Mats at right all represent zero.



Example 1

$3x + 2 - (2x - 3)$

Expressions can be simplified by moving tiles to the top (change the sign) and looking for zeros.



$3x + 2 - (2x - 3)$ $\quad$ $3x + 2 - 2x + 3$ $\quad$ $x + 5$

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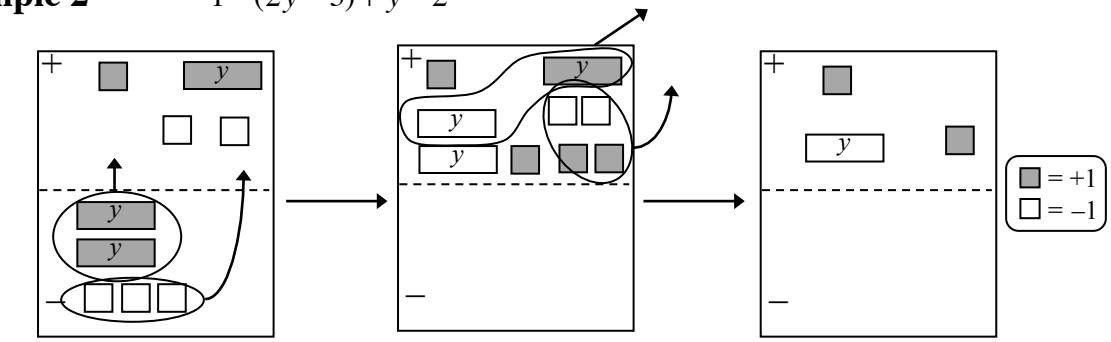
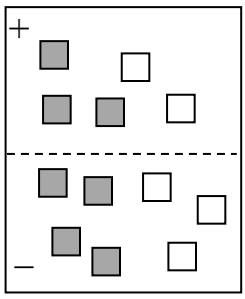
Example 2 $1-(2y-3)+y-2$

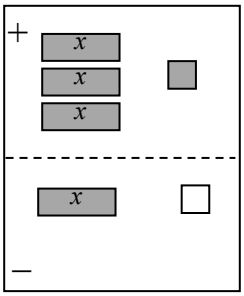


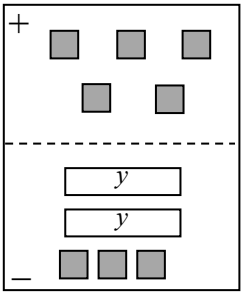
$1-(2y-3)+y-2$ $\quad$ $1-2y+3+y-2$ $\quad$ $-y+2$

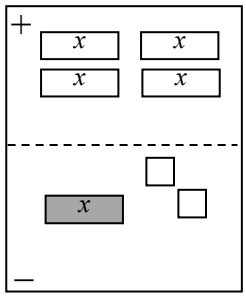
Problems

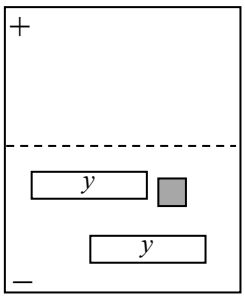
Simplify each expression.

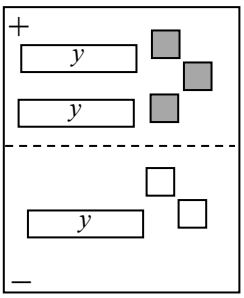
1. 

2. 

3. 

4. 

5. 

6. 

7. $3+5x-4-7x$
8. $-x-4x-7$
9. $-(-x+3)$
10. $4x-(x+2)$
11. $5x-(-3x+2)$
12. $x-5-(2-x)$
13. $1-2y-2y$
14. $-3x+5+5x-1$
15. $3-(y+5)$
16. $-(x+y)+4x+2y$
17. $3x-7-(3x-7)$
18. $-(x+2y+3)-3x+y$

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Answers

1.	0	2.	$2x + 2$	3.	$2y + 2$
4.	$-5x + 2$	5.	$2y - 1$	6.	$-y + 5$
7.	$-2x - 1$	8.	$-5x - 7$	9.	$x - 3$
10.	$3x - 2$	11.	$8x - 2$	12.	$2x - 7$
13.	$-4y + 1$	14.	$2x + 4$	15.	$-y - 2$
16.	$3x + y$	17.	0	18.	$-4x - y - 3$

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COMPARING QUANTITIES (ON AN EXPRESSION MAT) 2.1.5 – 2.1.8

Combining two Expression Mats into an Expression Comparison Mat creates a concrete model for simplifying (and later solving) inequalities and equations.

Tiles may be removed or moved on the mat in the following ways:

(1) Removing the same number of opposite tiles (zeros) on the same side;

(2) Removing an equal number of identical tiles (balanced set) from both the left and right sides;

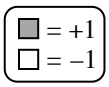
(3) Adding the same number of opposite tiles (zeros) on the same side; and

(4) Adding an equal number of identical tiles (balanced set) to both the left and right sides.

These strategies are called "legal moves."

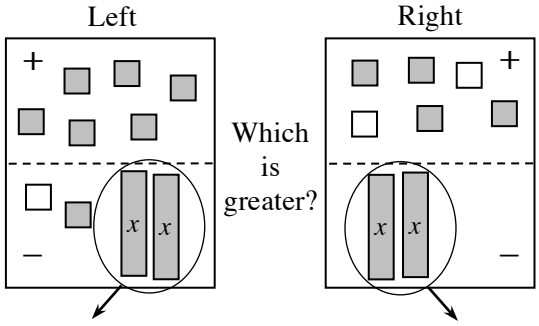
After moving and simplifying the Expression Comparison Mat, students are asked to tell which side is greater. Sometimes it is only possible to tell which side is greater if you know possible values of the variable.

Example 1

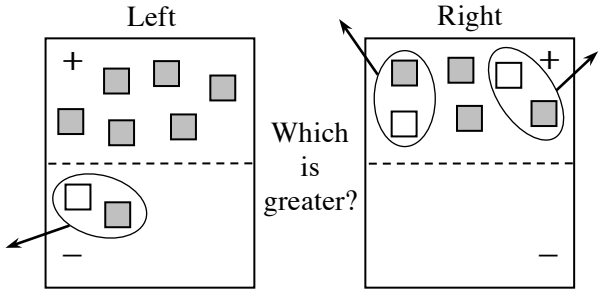


Determine which side is greater by using legal moves to simplify.

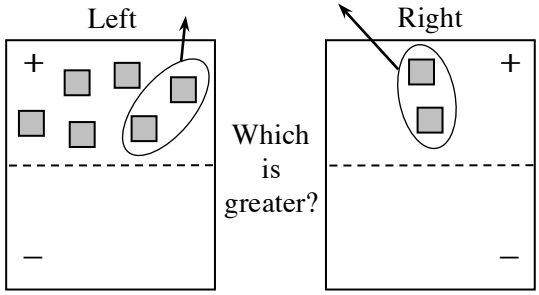
Step 1: Remove balanced set



Step 2: Remove zeros



Step 3: Remove balanced set



The left side is greater because after Step 3: $4 > 0$. Also, after Step 2: $6 > 2$. Note that this example shows only one of several possible strategies.

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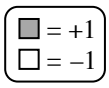
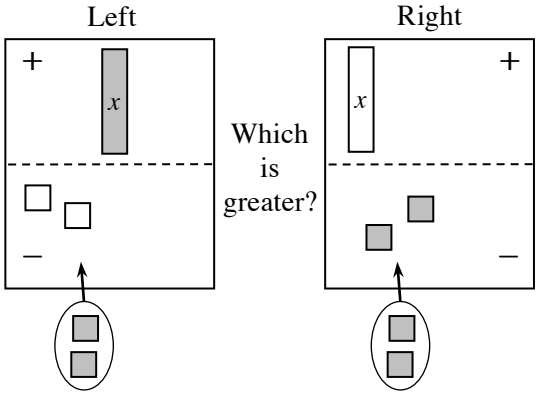
Example 2

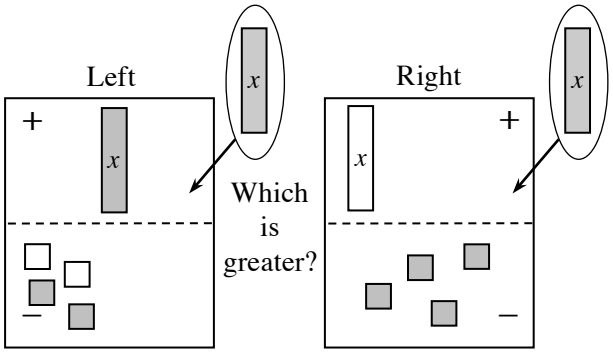


Use legal moves so that all the x-variables are on one side and all the unit tiles are on the other.

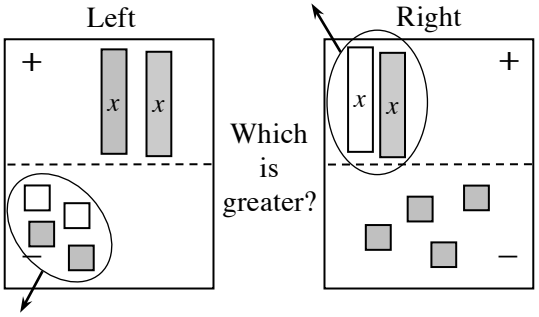
Step 1: Add balanced set



Step 2: Add balanced set



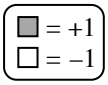
Step 3: Remove zeros



What remains is $2x$ on Mat A and -4 on Mat B. There are other possible arrangements. Whatever the arrangement, it is not possible to tell which side is greater because we do not know the value of "x." Students are expected to record the results algebraically as directed by the teacher. One possible recording is shown below.

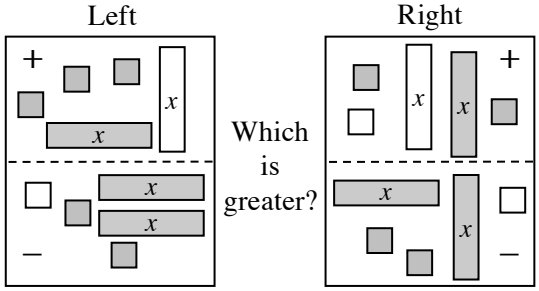
Left	Right
$x-(-2)$	$-x-(2)$
$x-(-2+2)$	$-x-(2+2)$
$x+x-(-2+2)$	$-x+x-(2+2)$
$2x$	4

Problems

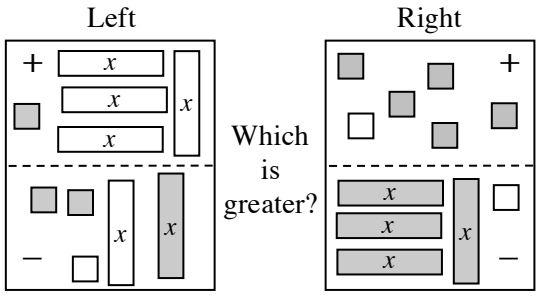


For each of the problems below, use the strategies of removing zeros or simplifying by removing balanced sets to determine which side is greater, if possible. Record your steps.

1.



2.



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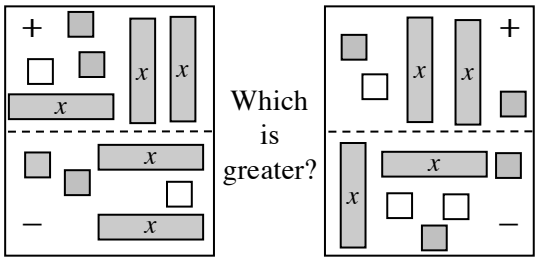
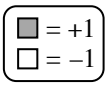
3. Left Right



4. Left: $5 + (-8)$
   Right: $-7 + 6$

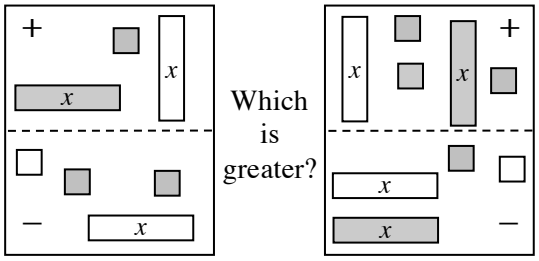


5. Left: $2(x + 3) - 2$
   Right: $4x - 2 - x + 4$

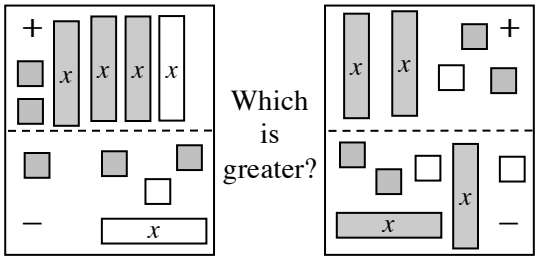
6. Left: $4 + (-2x) + 4x$
   Right: $x^2 + 2x + 3 - x^2$

For each of the problems below, use the strategies of removing zeros or adding/removing balanced sets so that all the x-variables are on one side and the unit tiles are on the other. Record your steps.

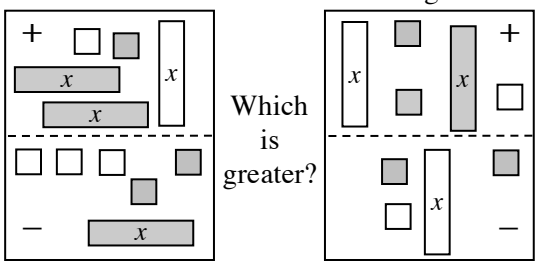
7. Left Right



8. Left Right



9. Left Right



10. Left: $3x - 2$
    Right: $2x + 1$

11. Left: $4x + 2 + (-5)$
    Right: $2x + 3 + (-8)$

12. Left: $2x + 3 + (-8)$
    Right: $-x - 3$

Answers (Answers to problems 7 through 12 may vary.)

1.	left	2.	right	3.	not possible to tell
4.	right	5.	not possible to tell	6.	left
7.	A: x; B: 3	8.	A: $3x$; B: 1	9.	A: 1; B: x
10.	A: x; B: 3	11.	A: $2x$; B: -2	12.	A: $3x$; B: -6

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Using a Four-Region Equation Mat

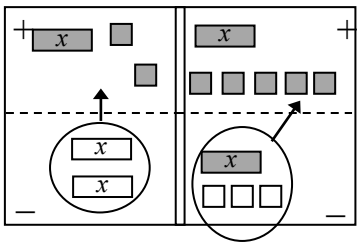
Combining two Expression Mats into an Equation Mat creates a concrete model for solving equations. Practice solving equations using the model will help students transition to solving equations abstractly with better accuracy and understanding.

In general, and as shown in the first example below, the negative in front of the parenthesis causes everything inside to "flip" from the top to the bottom or the bottom to the top of an Expression Mat, that is, all terms in the expression change signs. After simplifying the parentheses, simplify each Expression Mat. Next, isolate the variables on one side of the Equation Mat and the non-variables on the other side by removing matching tiles from both sides. Then determine the value of the variable. Students should be able to explain their steps. See the Math Notes boxes in Lessons 2.1.9 and 3.2.3 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 3* Checkpoint 5 materials.

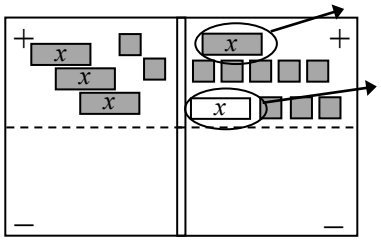
Procedure and Example

Solve $x + 2 - (-2x) = x + 5 - (x - 3)$.

First build the equation on the Equation Mat.

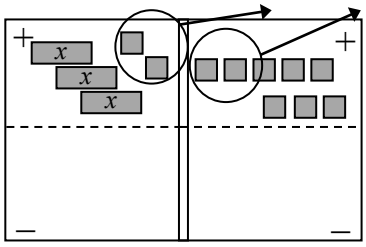


Second, simplify each side using legal moves on each Expression Mat, that is, on each side of the Equation Mat.



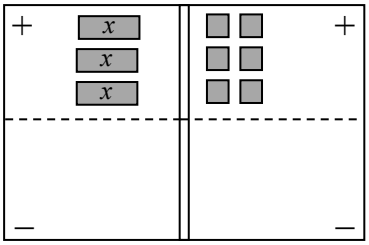
$x + 2 + 2x = x + 5 - x + 3$

Isolate x-terms on one side and non-x-terms on the other by removing matching tiles from both sides of the Equation Mat.



$3x + 2 = 8$

Finally, since both sides of the equation are equal, determine the value of x.



$3x = 6$

$x = 2$

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Once students understand how to solve equations using an Equation Mat, they may use the visual experience of moving tiles to solve equations with variables and numbers. The procedures for moving variables and numbers in the solving process follow the same rules.

Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, 2 = 4), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 2 = x + 2$) it means that *all numbers* are solutions. See the Math Notes box in Lesson 3.2.4 of the *Core Connections, Course 3* text.

Example 1 Solve: $3x + 3x - 1 = 4x + 9$

Solution:

$3x + 3x - 1 = 4x + 9$	problem
$6x - 1 = 4x + 9$	simplify
$2x = 10$	add 1, subtract $4x$ on each side
$x = 5$	divide

Example 2 Solve: $-2x + 1 - (-3x + 3) = -4 + (-x - 2)$

Solution:

$-2x + 1 - (-3x + 3) = -4 + (-x - 2)$	problem
$-2x + 1 + 3x - 3 = -4 - x - 2$	remove parenthesis (flip)
$x - 2 = -x - 6$	simplify
$2x = -4$	add x, add 2 to each side
$x = -2$	divide

Problems

Solve each equation.

1. $2x - 3 = -x + 3$
2. $1 + 3x - x = x - 4 + 2x$
3. $4 - 3x = 2x - 6$
4. $3 + 3x - (x - 2) = 3x + 4$
5. $-(x + 3) = 2x - 6$
6. $-4 + 3x - 1 = 2x + 1 + 2x$
7. $-x + 3 = 10$
8. $5x - 3 + 2x = x + 7 + 6x$
9. $4y - 8 - 2y = 4$
10. $9 - (1 - 3y) = 4 + y - (3 - y)$
11. $2x - 7 = -x - 1$
12. $-2 - 3x = x - 2 - 4x$
13. $-3x + 7 = x - 1$
14. $1 + 2x - 4 = -3 - (-x)$
15. $2x - 1 - 1 = x - 3 - (-5 + x)$
16. $-4x - 3 = x - 1 - 5x$
17. $10 = x + 6 + 2x$
18. $-(x - 2) = x - 5 - 3x$
19. $6 - x - 3 = 4x - 8$
20. $0.5x - (-x + 3) = x - 5$

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Answers

1.	$x = 2$	2.	$x = 5$	3.	$x = 2$	4.	$x = 1$	5.	$x = 1$
6.	$x = -6$	7.	$x = -7$	8.	no solution	9.	$x = 6$	10.	$x = -7$
11.	$x = 2$	12.	all numbers	13.	$x = 2$	14.	$x = 0$	15.	$x = 2$
16.	no solution	17.	$x = 1\frac{1}{3}$	18.	$x = -7$	19.	$x = 2\frac{1}{5}$	20.	$x = -4$

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TABLES, GRAPHS, AND RULES 3.1.1 – 3.1.7

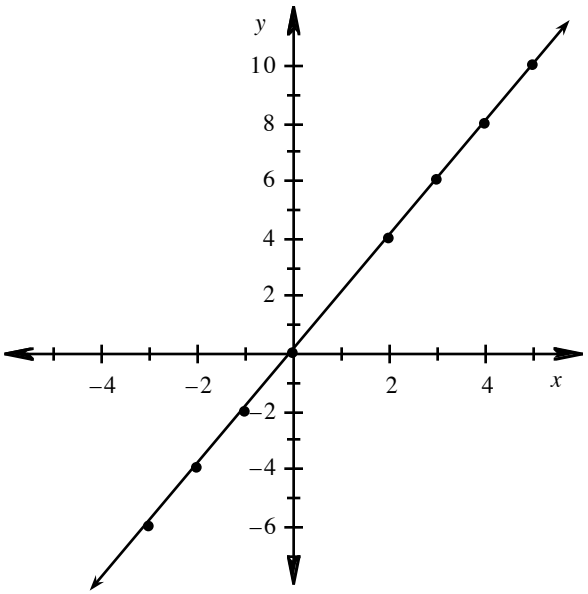
Three ways to write relationships for data are tables, words (descriptions), and rules. The pattern in tables between input (x) and output (y) values usually establishes the rule for a relationship. If you know the rule, it may be used to generate sets of input and output values. A description of a relationship may be translated into a table of values or a general rule (equation) that describes the relationship between the input values and output values. Each of these three forms of relationships may be used to create a graph to visually represent the relationship. For additional information, see the Math Notes boxes in Lessons 3.1.3, 3.1.4, 3.1.5, and 3.2.1 of the *Core Connections, Course 3* text.

Example 1

Complete the table by determining the relationship between the input (x) values and output (y) values, write the rule for the relationship, then graph the data.

input (x)	4	–3		5	0		3	–2	x
output (y)	8		4	10	0	–2		–4	

Begin by examining the four pairs of input values: 4 and 8, 5 and 10, 0 and 0, –2 and –4. Determine what arithmetic operation(s) are applied to the input value of each pair to get the second value. The operation(s) applied to the first value must be the *same* in all four cases to produce each given output value. In this example, the second value in each pair is twice the first value. Since the pattern works for all four points, make the conjecture that the rule is y (output) = $2x$ (input). This makes the missing values –3 and –6, 2 and 4, –1 and –2, 3 and 6. The rule is $y = 2x$. Finally, graph each pair of data on an xy-coordinate system, as shown at right.



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Example 2

Complete the table by determining the relationship between the input (x) and output (y) values, then write the rule for the relationship.

input (x)	2	–1	4	–3	0	–2	1	x
output (y)	3	–3	7	–7	–1	–5	1	

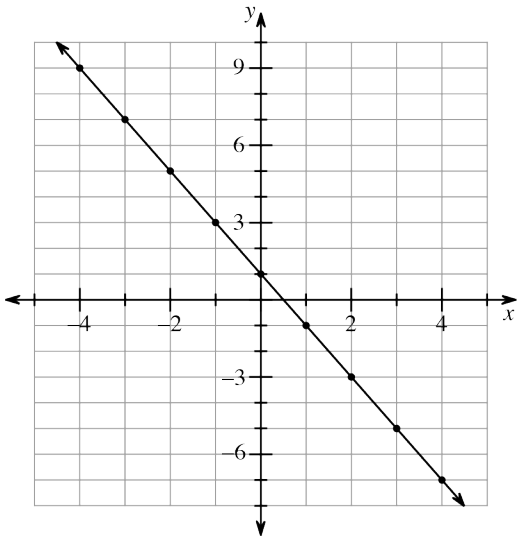
Use the same approach as Example 1. In this table, the relationship is more complicated than simply multiplying the input value or adding (or subtracting) a number. Use a Guess and Check approach to try different patterns. For example, the first pair of values could be found by the rule $x + 1$, that is, $2 + 1 = 3$. However, that rule fails when you check it for –1 and –3: $–1 + 1 \neq –3$. From this guess you know that the rule must be some combination of multiplying the input value and then adding or subtracting to that product. The next guess could be to double x. Try it for the first two or three input values and see how close each result is to the known output values: for 2 and 3, $2(2) = 4$; for –1 and –3, $2(–1) = –2$; and for 4 and 7, $2(4) = 8$. Notice that each result is one more than the actual output value. If you subtract 1 from each product, the result is the expected output value. Make the conjecture that the rule is y (output) = $2x$ (input) – 1 and test it for the other input values: for –3 and –7, $2(–3) – 1 = –7$; for 0 and –1, $2(0) – 1 = –1$; for –2 and –5, $2(–2) – 1 = –5$; and for 1 and 1, $2(1) – 1 = 1$. So the rule is $y = 2x – 1$.

Example 3

Complete the table below for $y = –2x + 1$, then graph each of the points in the table.

input (x)	–4	–3	–2	–1	0	1	2	3	4	x
output (y)										

Replace x with each input value, multiply by –2, then add 1. The results are ordered pairs: (–4, 9), (–3, 7), (–2, 5), (–1, 3), (0, 1), (1, –1), (2, –3), (3, –5), and (4, –7). Plot these points on the graph (see Four-Quadrant Graphing if you need help with the fundamentals of graphing).



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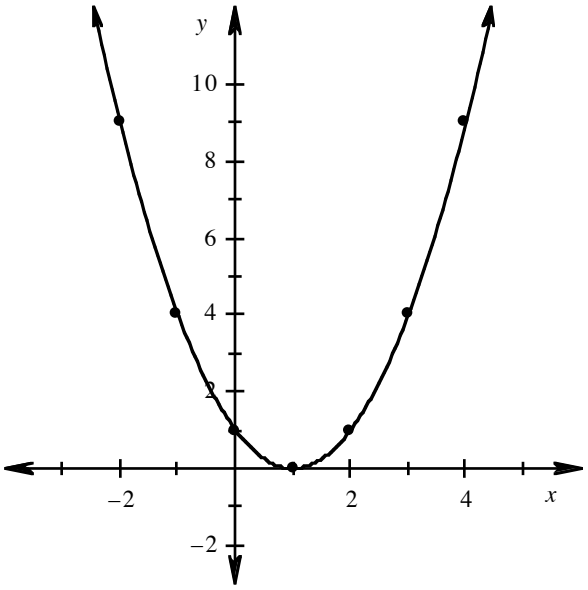
Example 4

Complete the table below for $y = x^2 - 2x + 1$, then graph the pairs of points and connect them with a smooth curve.

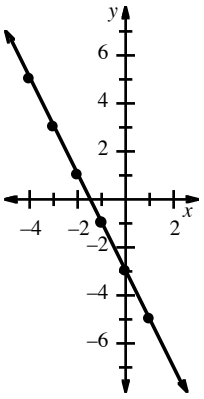
input (x)	–2	–1	0	1	2	3	4	x
output (y)								$x^2 - 2x + 1$

Replace x in the equation with each input value. Square the value, multiply the value by –2, then add both of these results and 1 to get the output (y) value for each input (x) value. The results are ordered pairs: (–2, 9), (–1, 4), (0, 1), (1, 0), (2, 1), (3, 4), and (4, 9).



Example 5

Make an $x \rightarrow y$ table for the graph at right, then write a rule for the table.



input (x)	–4	–3	–2	–1	0	1	x
output (y)	5	3	1	–1	–3	–5	$-2x - 3$

Working left to right on the graph, read the coordinates of each point and record them in the table.

Guess and check by multiplying the input value, then adding or subtracting numbers to get the output value. For example you could start by multiplying the input value by 2: $2(-4) = -8$, $3(-4) = -12$, $2(-3) = -6$, etc. The results are not close to the correct output value. The product is also the opposite sign (+ –) of what you want. Your next choice could be to multiply by –2: $-2(-4) = 8$, $-2(-3) = 6$, $-2(-2) = 4$. Each result is three more than the expected output value, so make the conjecture that the rule is $y = -2x - 3$. Test it for the remaining points: $-2(-1) - 3 = -1$, $-2(0) - 3 = -3$, and $-2(1) - 3 = -5$. The rule is $y = -2x - 3$.

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Problems

Complete each table. Then write a rule relating x and y.

1.

input (x)	10	5	20		–7	3	x
output (y)	14	9		–4	–3		

2.

input (x)	22	5	11		–9	12	x
output (y)	19	2		–8	–12		

3.

input (x)	10	0	15		–7	6	x
output (y)	25	–5		–8		13	

4.

input (x)	10	–3	4		–5	15	x
output (y)	–30	9	–12	–1			

5.

input (x)	–4	0	12		–16		x
output (y)	0	2		–4	–6	6	

6.

input (x)	4	–6		2	–12		x
output (y)	7	–3	6		–9	1	

7.

input (x)	3	0		–12	16	9	x
output (y)	3	–3	13		29		

8.

input (x)	2	0	–4		–13		x
output (y)	–5	1		–20	40	–17	

9.

input (x)	9	–6		0	3	–3	x
output (y)	3	–2	4		1		

10.

input (x)	–3	–2	–1	0	1	2	x
output (y)	–7	–4	–1	2	5	8	

11.

input (x)	–6	–4	–2	0	2	4	x
output (y)	1	2	3	4	5	6	

12.

input (x)	–2	–1	0	1	2	3	x
output (y)	5	2	1	2	5	10	

Complete a table for each rule, then graph and connect the points. For each rule, start with a table like the one below.

input (x)	–3	–2	–1	0	1	2	3
output (y)							

13. $y = 3x - 2$ 14. $y = \frac{1}{2}x + 1$ 15. $y = -x + 2$ 16. $y = x^2 - 6$

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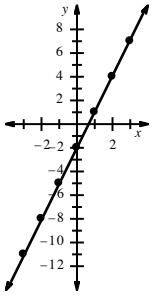
Answers

1. $24, -8, 7$; $y = x + 4$
2. $8, -5, 9$; $y = x - 3$
3. $40, -1, -26$; $y = 3x - 5$
4. $\frac{1}{3}, 15, -45$; $y = -3x$
5. $8, -12, 8$; $y = -\frac{1}{2}x + 2$
6. $3, 5, -2$; $y = x + 3$
7. $8, -27, 15$; $y = 2x - 3$
8. $13, 7, 6$; $y = -3x + 1$
9. $12, 0, -1$; $y = \frac{x}{3}$
10. $y = 3x + 2$
11. $y = \frac{1}{2}x + 4$
12. $y = x^2 + 1$

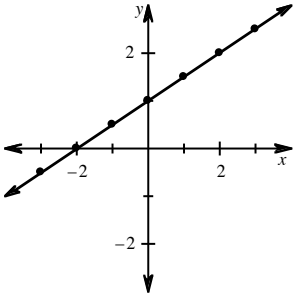
13.

input (x)	–3	–2	–1	0	1	2	3
output (y)	–11	–8	–5	–2	1	4	7



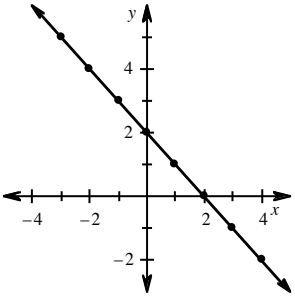
14.

input (x)	–3	–2	–1	0	1	2	3
output (y)	–0.5	0	0.5	1	1.5	2	2.5



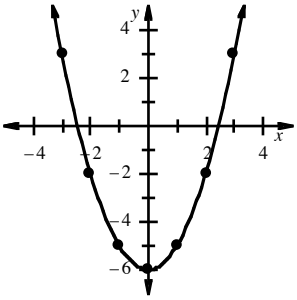
15.

input (x)	–3	–2	–1	0	1	2	3
output (y)	5	4	3	2	1	0	–1



16.

input (x)	–3	–2	–1	0	1	2	3
output (y)	3	–2	–5	–6	–5	–2	3



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DISTRIBUTIVE PROPERTY 3.2.5

The Distributive Property shows how to express sums and products in two ways: $a(b+c)=ab+ac$. This can also be written $(b+c)a=ab+ac$.

Factored form	Distributed form	Simplified form
$\boldsymbol{a(b+c)}$	$\boldsymbol{a(b)+a(c)}$	$\boldsymbol{ab+ac}$

To simplify: Multiply each term on the inside of the parentheses by the term on the outside. Combine terms if possible.

For additional information, see the Math Notes box in Lesson 3.2.5 of the *Core Connections, Course 3* text.

Example 1

$$\begin{aligned} 2(47) &= 2(40+7) \\ &= (2\cdot 40)+(2\cdot 7) \\ &= 80+14=94 \end{aligned}$$

Example 2

$$\begin{aligned} 3(x+4) &= (3\cdot x)+(3\cdot 4) \\ &= 3x+12 \end{aligned}$$

Example 3

$$\begin{aligned} 4(x+3y+1) &= (4\cdot x)+(4\cdot 3y)+4(1) \\ &= 4x+12y+4 \end{aligned}$$

Problems

Simplify each expression below by applying the Distributive Property.

1. $6(9+4)$
2. $4(9+8)$
3. $7(8+6)$
4. $5(7+4)$
5. $3(27)=3(20+7)$
6. $6(46)=6(40+6)$
7. $8(43)$
8. $6(78)$
9. $3(x+6)$
10. $5(x+7)$
11. $8(x-4)$
12. $6(x-10)$
13. $(8+x)4$
14. $(2+x)5$
15. $-7(x+1)$
16. $-4(y+3)$
17. $-3(y-5)$
18. $-5(b-4)$
19. $-(x+6)$
20. $-(x+7)$
21. $-(x-4)$
22. $-(-x-3)$
23. $x(x+3)$
24. $4x(x+2)$
25. $-x(5x-7)$
26. $-x(2x-6)$

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Answers

1. $(6 \cdot 9) + (6 \cdot 4) = 54 + 24 = 78$
2. $(4 \cdot 9) + (4 \cdot 8) = 36 + 32 = 68$
3. $56 + 42 = 98$
4. $35 + 20 = 55$
5. $60 + 21 = 81$
6. $240 + 36 = 276$
7. $320 + 24 = 344$
8. $420 + 48 = 468$
9. $3x + 18$
10. $5x + 35$
11. $8x - 32$
12. $6x - 60$
13. $4x + 32$
14. $5x + 10$
15. $-7x - 7$
16. $-4y - 12$
17. $-3y + 15$
18. $-5b + 20$
19. $-x - 6$
20. $-x - 7$
21. $-x + 4$
22. $x + 3$
23. $x^2 + 3x$
24. $4x^2 + 8x$
25. $-5x^2 + 7x$
26. $-2x^2 + 6x$

When the Distributive Property is used to reverse, it is called factoring. Factoring changes a sum of terms (no parentheses) to a product (with parentheses).

$$ab + ac = a(b + c)$$

To factor: Write the common factor of all the terms outside of the parentheses. Place the remaining factors of each of the original terms inside of the parentheses.

Example 4

$$\begin{aligned} 4x + 8 &= 4 \cdot x + 4 \cdot 2 \\ &= 4(x + 2) \end{aligned}$$

Example 5

$$\begin{aligned} 6x^2 - 9x &= 3x \cdot 2x - 3x \cdot 3 \\ &= 3x(2x - 3) \end{aligned}$$

Example 6

$$\begin{aligned} 6x + 12y + 3 &= 3 \cdot 2x + 3 \cdot 4y + 3 \cdot 1 \\ &= 3(2x + 4y + 1) \end{aligned}$$

Problems

Factor each expression below by using the Distributive Property in reverse.

1. $6x + 12$
2. $5y - 10$
3. $8x + 20z$
4. $x^2 + xy$
5. $8m + 24$
6. $16y + 40$
7. $8m - 2$
8. $25y - 10$
9. $2x^2 - 10x$
10. $21x^2 - 63$
11. $21x^2 - 63x$
12. $15y + 35$
13. $4x + 4y + 4z$
14. $6x + 12y + 6$
15. $14x^2 - 49x + 28$
16. $x^2 - x + xy$

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Answers

1.	$6(x+2)$	2.	$5(y-2)$	3.	$4(2x+5z)$	4.	$x(x+y)$
5.	$8(m+3)$	6.	$8(2y+5)$	7.	$2(4m-1)$	8.	$5(5y-2)$
9.	$2x(x-5)$	10.	$21(x^2-3)$	11.	$21x(x-3)$	12.	$5(3y+7)$
13.	$4(x+y+z)$	14.	$6(x+2y+1)$	15.	$7(2x^2-7x+4)$	16.	$x(x-1+y)$

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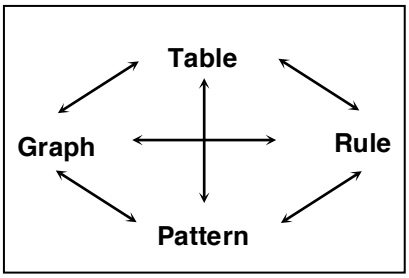
The first part of Chapter 4 of *Core Connections, Course 3* ties together several ways to represent the same relationship. The basis for any relationship is a consistent pattern that connects input and output values. This course uses tile patterns to help visualize algebraic relationships. (Note: In this course we consider tile patterns to be continuous relationships and graph them with a continuous line or curve.) These relationships may also be displayed on a graph, in a table, or as an equation. In each situation, all four representations show the same relationship. Students learn how to use each format to display relationships as well as how to switch from one representation to another. We use the diagram at right to show the connections between the various ways to display a relationship and call it the "representations web." See the Math Notes box in Lesson 4.1.7 of the *Core Connections, Course 3* text. For additional examples and practice see the *Core Connections, Course 3* Checkpoint 6 materials.



Example 1

At this point in the course we use the notion of growth to help understand linear relationships. For example, a simple tile pattern may start with two tiles and grow by three tiles in each successive figure as shown below.

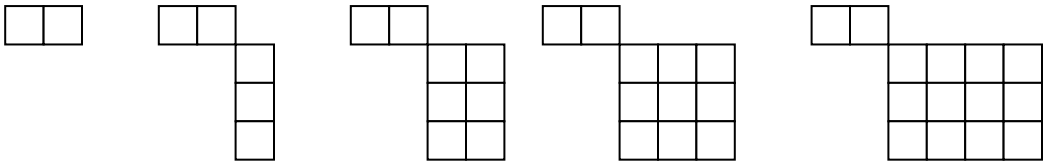
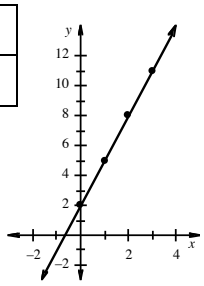


Fig. 0 Fig. 1 Fig. 2 Fig. 3 Fig. 4

The picture of the tile figures may also be described by an equation in $y = mx + b$ form, where x and y are variables and m represents the growth rate and b represents the starting value of the pattern. In this example, $y = 3x + 2$, where 2 represents the number of tiles in the original figure (usually called "Figure 0") and 3 is the "growth factor" that describes the rate at which each successive figure adds tiles to the previous figure. This relationship may also be displayed in a table, called an "$x \rightarrow y$ table," as shown below. The rule is written in the last column of the table.

Figure number (x)	0	1	2	3	4	x
Number of tiles (y)	2	5	8	11	14	$3x + 2$

Finally, the relationship may be displayed on an xy-coordinate graph by plotting the points in the table as shown at right. The highlighted points on the graph represent the tile pattern. The line represents all of the points described by the equation $y = 3x + 2$.



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Example 2

Draw Figures 0, 4, and 5 for the tile pattern below. Use the pattern to predict the number of tiles in Figure 100, describe the figure, write a rule that will give the number of tiles in any figure, record the data for the first six tiles (Figures 0 through 5) in a table, and graph the data.

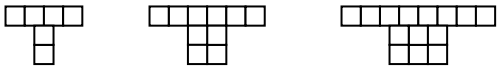
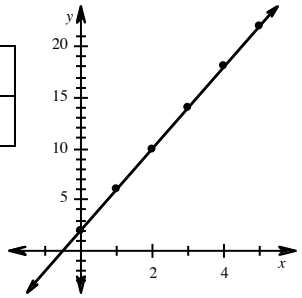


Fig. 0 Fig. 1 Fig. 2 Fig. 3 Fig. 4 Fig. 5

Each figure adds four tiles: two tiles to the top row and two tiles to the lower portion of the figure. Figure 0 has two tiles, so the rule is $y = 4x + 2$ and Figure 100 has $4(100) + 2 = 402$ tiles. There are 202 tiles in the top row and 200 tiles in the lower portion of figure 100. The table is:

Figure number (x)	0	1	2	3	4	5	x
Number of tiles (y)	2	6	10	14	18	22	$4x + 2$

The graph is shown at right.



Example 3

Use the table below to determine the rule in $y = mx + b$ form that describes the pattern.

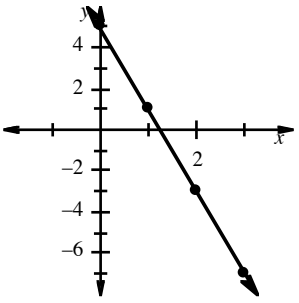
input (x)	–2	–1	0	1	2	3	4	5	x
output (y)	–8	–5	–2	1	4	7	10	13	

+3 +3 +3 +3 +3 +3 +3

The constant difference between the output values is the growth rate, that is, the value of m. The output value paired with the input value $x = 0$ is the starting value, that is, the value of b. So this table can be described by the rule: $y = 3x - 2$. Note: If there is not a constant difference between the output values for consecutive integer input values, then the rule for the pattern is not in the form $y = mx + b$.

Example 4

Use the graph at right to create an $x \rightarrow y$ table, then write a rule for the pattern it represents.



First transfer the coordinates of the points into an $x \rightarrow y$ table.

input (x)	0	1	2	3	x
output (y)	5	1	–3	–7	

Using the method described in Example 3, that is, noting that the growth rate between the output values is –4 and the value of y at $x = 0$ is 5, the rule is: $y = -4x + 5$.

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Problems

1. Based on the tile pattern below, draw Figures 0, 4, and 5. Then find a rule that will give the number of tiles in any figure and use it to find the number of tiles in Figure 100. Finally, display the data for the first six figures (numbers 0-5) in a table and on a graph.

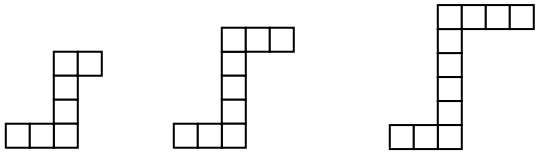


Fig. 0 Fig. 1 Fig. 2 Fig. 3 Fig. 4 Fig. 5

2. Based on the tile pattern below, draw Figures 0, 4, and 5. Then find a rule that will give the number of tiles in any figure and use it to find the number of tiles in Figure 100. Finally, display the data for the first six figures (numbers 0-5) in a table and on a graph.

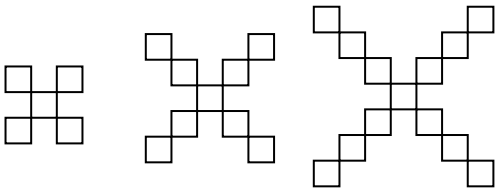


Fig. 0 Fig. 1 Fig. 2 Fig. 3 Fig. 4 Fig. 5

Use the patterns in the tables and graphs to write rules for each relationship.

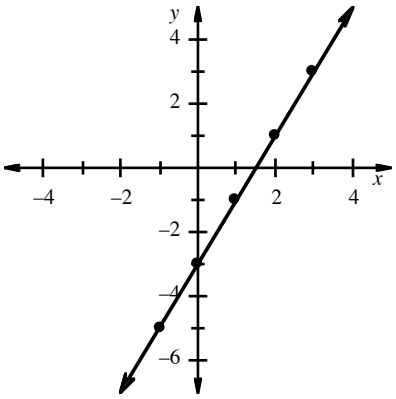
3.

input (x)	–3	–2	–1	0	1	2	3	4	5
output (y)	–11	–8	–5	–2	1	4	7	10	13

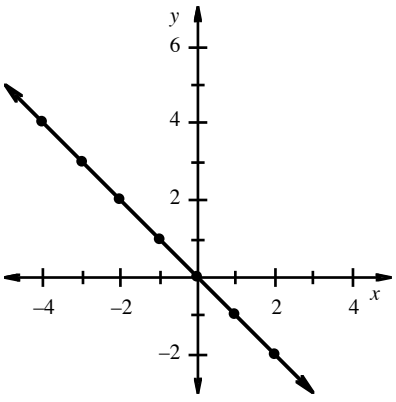
4.

input (x)	–3	–2	–1	0	1	2	3	4	5
output (y)	10	8	6	4	2	0	–2	–4	–6

5.



6.



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Answers

1.

Fig. 0

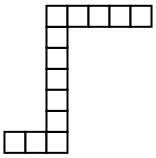


Fig. 4

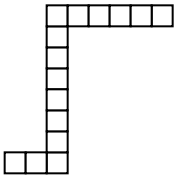
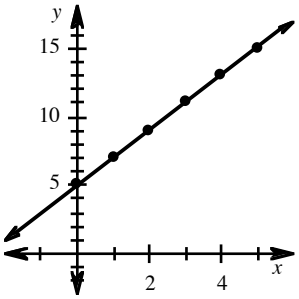


Fig. 5



The rule is $y = 2x + 5$. Figure 100 will have 205 tiles. It will have a base of three tiles, with 102 tiles extending up from the right tile in the base and 100 tiles extending to the right of the top tile in the vertical extension above the base.

Figure number (x)	0	1	2	3	4	5
Number of tiles (y)	5	7	9	11	13	15

2.

Fig. 0

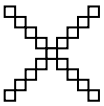


Fig. 4

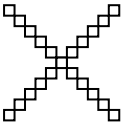
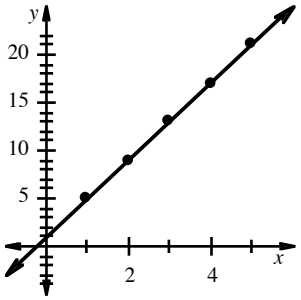


Fig. 5



The rule is $y = 4x + 1$. Figure 100 will have 401 tiles in the shape of an "X" with 100 tiles on each "branch" of the X, all connected to a single square in the middle.

Figure number (x)	0	1	2	3	4	5
Number of tiles (y)	1	5	9	13	17	21

3. $y = 3x - 2$

4. $y = -2x + 4$

5. $y = 2x - 3$

6. $y = -x$

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LINEAR GRAPHS USING $y = mx + b$ 4.1.4 – 4.1.7

Slope (rate of change) is a number that indicates the steepness (or flatness) of a line, that is, its rate of change, as well as its direction (up or down) left to right.

Slope (rate of change) is determined by the ratio: $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x}$

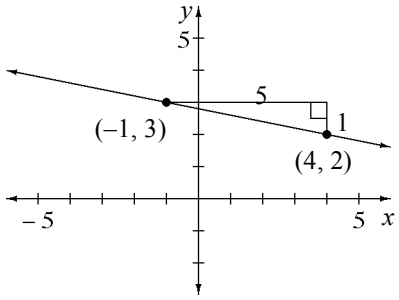
between *any* two points on a line. Some books and teachers refer to this ratio as the rise (y) over the run (x).

For lines that go up (from left to right), the sign of the slope is positive. For lines that go down (left to right), the sign of the slope is negative.

Any linear equation written as $\boldsymbol{y = mx + b}$, where m and b are any real numbers, is in slope-intercept form. m is the slope of the line. b is the y-intercept, that is, the point $(0, b)$ where the line intersects (crosses) the y-axis.

Example 1

Write the slope of the line containing the points (–1, 3) and (4, 2).



First graph the two points and draw the line through them.

Look for and draw a slope triangle using the two given points.

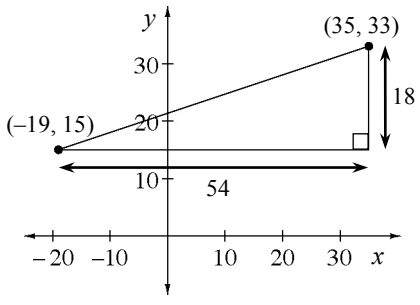
Write the ratio $\frac{\text{vertical change in } y}{\text{horizontal change in } x}$ using the legs of the right triangle: $\frac{1}{5}$.

Assign a positive or negative value to the slope depending on whether the line goes up (+) or down (–) from left to right. The slope is $-\frac{1}{5}$.

Example 2

Write the slope of the line containing the points (–19, 15) and (35, 33).

Since the points are inconvenient to graph, use a "generic slope triangle," visualizing where the points lie with respect to each other and the axes.



Make a sketch of the points.

Draw a slope triangle and determine the length of each leg. Write the ratio of y to x: $\frac{18}{54} = \frac{1}{3}$. The slope is $\frac{1}{3}$.

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Example 3

Given a table, determine the rate of change (slope) and the equation of the line.

	+2	+2	+2	
x	–2	0	2	4
y	1	4	7	9
	+3	+3	+3	

rate of change = $\frac{3}{2}$

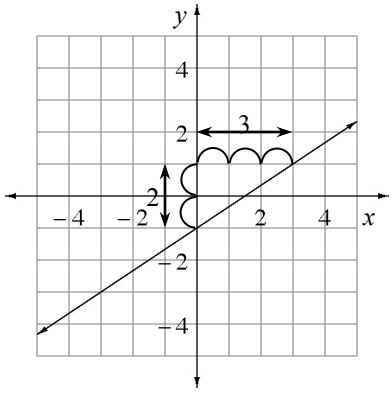
y-intercept = (0, 4)

The equation of the line is $y = \frac{3}{2}x + 4$.

Example 4

Graph the linear equation $y = \frac{2}{3}x - 1$.

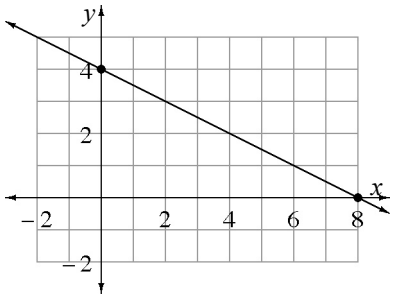
Using $y = mx + b$, the slope in $y = \frac{2}{3}x - 1$ is $\frac{2}{3}$ and the y-intercept is the point (0, –1). To graph, begin at the y-intercept (0, –1). Remember that slope is $\frac{\text{vertical change}}{\text{horizontal change}}$, so go up 2 units (since 2 is positive) from (0, –1) and then move right 3 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.



Problems

Determine the slope of each line using the highlighted points.

1.



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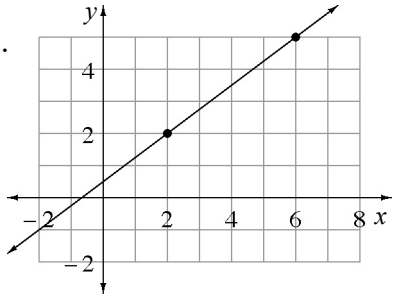
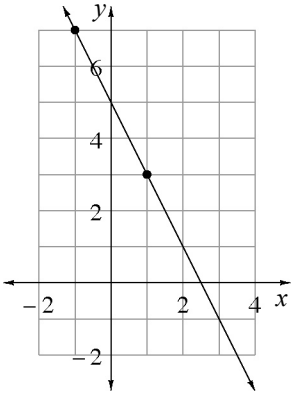


3.



Find the slope of the line containing each pair of points. Sketch a slope triangle to visualize the vertical and horizontal change.

4. (2, 3) and (5, 7)
5. (2, 5) and (9, 4)
6. (1, –3) and (7, –4)
7. (–2, 1) and (3, –3)
8. (–2, 5) and (4, 5)
9. (5, 8) and (3, 5)

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Use a slope triangle to find the slope of the line containing each pair of points:

10. (50, 40) and (30, 75)

11. (10, 39) and (44, 80)

12. (5, –13) and (–51, 10)

Identify the slope and *y*-intercept in each equation.

13. $y = \frac{1}{2}x - 2$

14. $y = -3x + 5$

15. $y = 4x$

16. $y = -\frac{2}{3}x + 1$

17. $y = x - 7$

18. $y = 5$

Draw a graph to find the equation of the line with:

19. slope = $\frac{1}{2}$ and passing through (2, 3).

20. slope = $\frac{2}{3}$ and passing through (3, –2).

21. slope = $-\frac{1}{3}$ and passing through (3, –1).

22. slope = –4 and passing through (–3, 8).

For each table, determine the rate of change and the equation. Be sure to record whether the rate of change is positive or negative for *both* x and y.

23.

x	–2	–1	0	1	2
y	–5	–2	1	4	7

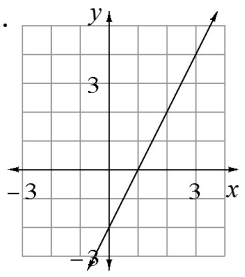
24.

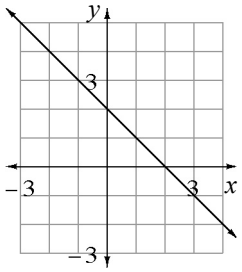
x	–2	0	2	4	6
y	7	3	–1	–5	–9

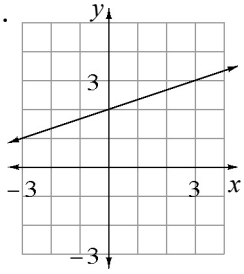
25.

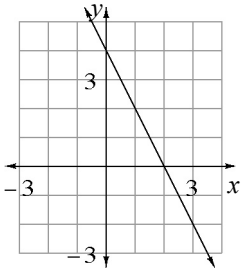
x	–6	–3	0	3	6
y	–3	–1	1	3	5

Using the slope and *y*-intercept, determine the equation of the line.

26. 

27. 

28. 

29. 

Graph the following linear equations on graph paper.

30. $y = \frac{1}{2}x + 2$

31. $y = -\frac{3}{5}x + 1$

32. $y = -4x$

33. $y = 2x + \frac{1}{2}$

34. $3x + 2y = 12$

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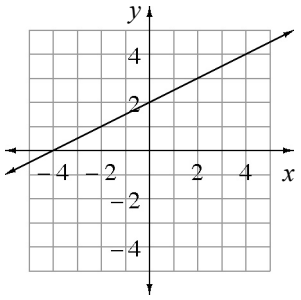
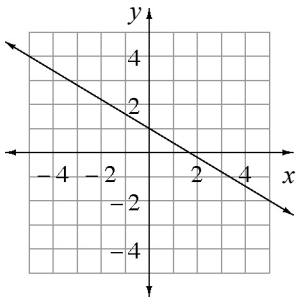
Answers

1. $-\frac{1}{2}$
2. $\frac{3}{4}$
3. -2
4. $\frac{4}{3}$
5. $-\frac{1}{7}$
6. $\frac{1}{6}$
7. $-\frac{4}{5}$
8. 0
9. $\frac{3}{2}$
10. $-\frac{35}{20}=-\frac{7}{4}$
11. $\frac{41}{34}$
12. $-\frac{33}{71}$
13. $\frac{1}{2}$; $(0,-2)$
14. -3; $(0,-5)$
15. 4; $(0,0)$
16. $-\frac{2}{3}$; $(0,1)$
17. 1; $(0,-7)$
18. 0; $(0,5)$
19. $y=\frac{1}{2}x+2$
20. $y=\frac{2}{3}x-4$
21. $y=-\frac{1}{3}$
22. $y=-4x-4$
23. 3; $y=3x+1$
24. -2; $y=-2x+3$
25. $\frac{2}{3}$; $y=\frac{2}{3}x+1$
26. $y=2x-2$
27. $y=-x+2$
28. $y=\frac{1}{3}x+2$
29. $y=-2x+4$

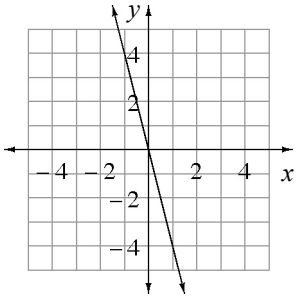
30. $y=\frac{1}{2}x+2$



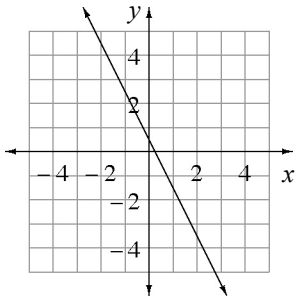
31. $y=-\frac{3}{5}x+1$



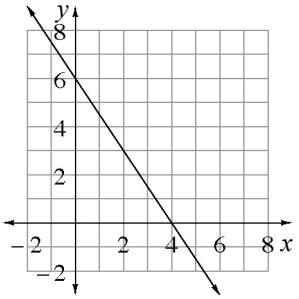
32. $y=-4x$



33. $y=-2x+\frac{1}{2}$



34. $y=-\frac{3}{2}x+6$



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EQUATIONS WITH MULTIPLE VARIABLES 5.1.1

Solving equations with more than one variable uses the same process as solving an equation with one variable. The only difference is that instead of the answer always being a number, it may be an expression that includes numbers and variables. The usual steps may include: removing parentheses, simplifying by combining like terms, removing the same thing from both sides of the equation, moving the desired variables to one side of the equation and the rest of the variables to the other side, and possibly division or multiplication.

Example 1

Solve for y $3x - 2y = 6$

Subtract $3x$ $-2y = -3x + 6$

Divide by -2 $y = \frac{-3x+6}{-2}$

Simplify $y = \frac{3}{2}x - 3$

Example 2

Solve for y $7 + 2(x + y) = 11$

Subtract 7 $2(x + y) = 4$

Distribute the 2 $2x + 2y = 4$

Subtract $2x$ $2y = -2x + 4$

Divide by 2 $y = \frac{-2x+4}{2}$

Simplify $y = -x + 2$

Example 3

Solve for x $y = 3x - 4$

Add 4 $y + 4 = 3x$

Divide by 3 $\frac{y+4}{3} = x$

Example 4

Solve for t $I = prt$

Divide by pr $\frac{I}{pr} = t$

Problems

Solve each equation for the specified variable.

1. y in $5x + 3y = 15$
2. x in $5x + 3y = 15$
3. w in $2l + 2w = P$
4. m in $4n = 3m - 1$
5. a in $2a + b = c$
6. a in $b - 2a = c$
7. p in $6 - 2(q - 3p) = 4p$
8. x in $y = \frac{1}{4}x + 1$
9. r in $4(r - 3s) = r - 5s$

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Answers (Other equivalent forms are possible.)

1. $y = -\frac{5}{3}x + 5$
2. $x = -\frac{3}{5}y + 3$
3. $w = -l + \frac{P}{2}$
4. $m = \frac{4n+1}{3}$
5. $a = \frac{c-b}{2}$
6. $a = \frac{c-b}{-2}$ *or* $\frac{b-c}{2}$
7. $p = q - 3$
8. $x = 4y - 4$
9. $r = \frac{7s}{3}$

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EQUATIONS WITH FRACTIONAL COEFFICIENTS 5.1.2

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving one-step equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient.

To remove fractions in more complicated equations students use "Fraction Busters." Multiplying all of the terms of an equation by the common denominator will remove all of the fractions from the equation. Then the equation can be solved in the usual way.

For additional information, see the Math Notes box in Lesson 5.2.1 of the *Core Connections, Course 3* text. For additional examples and practice see the *Core Connections, Course 3* Checkpoint 7 materials.

Example of a One-Step Equation

Solve: $\frac{2}{3}x = 12$

Method 1: Use division and common denominators

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3}x = 12$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(12)$$

$$x = 18$$

Example of Fraction Busters

Solve: $\frac{x}{2} + \frac{x}{5} = 6$

Multiplying by 10 (the common denominator) will eliminate the fractions.

$$10\left(\frac{x}{2} + \frac{x}{5}\right) = 10(6)$$

$$10\left(\frac{x}{2}\right) + 10\left(\frac{x}{5}\right) = 10(6)$$

$$5x + 2x = 60$$

$$7x = 60 \Rightarrow x = \frac{60}{7} \approx 8.57$$

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Problems

Solve each equation.

1. $\frac{3}{4}x = 60$
2. $\frac{2}{5}x = 42$
3. $\frac{3}{5}y = 40$
4. $-\frac{8}{3}m = 6$
5. $\frac{3x+1}{2} = 5$
6. $\frac{x}{3} - \frac{x}{5} = 3$
7. $\frac{y+7}{3} = \frac{y}{5}$
8. $\frac{m}{3} - \frac{2m}{5} = \frac{1}{5}$
9. $-\frac{3}{5}x = \frac{2}{3}$
10. $\frac{x}{2} + \frac{x-3}{5} = 3$
11. $\frac{1}{3}x + \frac{1}{4}x = 4$
12. $\frac{2x}{5} + \frac{x-1}{3} = 4$

Answers

1. $x = 80$
2. $x = 105$
3. $y = 66\frac{2}{3}$
4. $m = -\frac{9}{4}$
5. $y = 3$
6. $x = 22.5$
7. $y = -17\frac{1}{2}$
8. $m = -3$
9. $x = -\frac{10}{9}$
10. $x = \frac{36}{7}$
11. $x = \frac{48}{7}$
12. $x = \frac{65}{11}$

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SYSTEMS OF LINEAR EQUATIONS 5.2.1 – 5.2.4

Two lines on an *xy*-coordinate grid are called a system of linear equations. They intersect at a point unless they are parallel or the equations are different forms of the same line. The point of intersection is the only pair of (*x*, *y*) values that will make *both* equations true. One way to find the point of intersection is to graph the two lines. However, graphing is both time-consuming and, in many cases, not exact, because the result may only be a close approximation of the coordinates.

When two linear equations are written equal to y (in general, in the form $y = mx + b$), we can take advantage of the fact that both *y* values are the same (equal) at the point of intersection. For example, if two lines are described by the equations $y = -2x + 5$ and $y = x - 1$, and we know that both *y* values are equal, then the other two sides of the equations must also be equal to each other. We say that both right sides of these equations have "equal values" at the point of intersection and write $-2x + 5 = x - 1$, so that the result looks like the work we did with equation mats.

We can solve this equation in the usual way and find that $x = 2$. Now we know the *x*-coordinate of the point of intersection. Since this value will be the same in both of the original equations at the point of intersection, we can substitute $x = 2$ in either equation to solve for *y*: $y = -2(2) + 5$ so $y = 1$ or $y = 2 - 1$ and $y = 1$. So the two lines in this example intersect at (2, 1).

For additional information, see the Math Notes boxes in Lessons 5.2.2, 5.2.3, and 5.2.4 of the *Core Connections, Course 3* text.

Example 1

Find the point of intersection for $y = 5x + 1$ and $y = -3x - 15$.

Substitute the equal parts of the equations. $5x + 1 = -3x - 15$

Solve for *x*. $8x = -16$

$x = -2$

Replace x with -2 in either original equation and solve for *y*.

$y = 5(-2) + 1$ or $y = -3(-2) - 15$

$y = -10 + 1$ or $y = 6 - 15$

$y = -9$ $y = -9$

The two lines intersect at (–2, –9).

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Example 2

The Mathematical Amusement Park is different from other amusement parks. Visitors encounter their first decision involving math when they pay their entrance fee. They have a choice between two plans. With Plan 1 they pay \$5 to enter the park and \$3 for each ride. With Plan 2 they pay \$12 to enter the park and \$2 for each ride. For what number of rides will the plans cost the same amount?

The first step in the solution is to write an equation that describes the total cost of each plan. In this example, let x equal the number of rides and y be the total cost. Then the equation to represent Plan 1 for x rides is $y = 5 + 3x$. Similarly, the equation representing Plan 2 for x rides is $y = 12 + 2x$.

We know that if the two plans cost the same, then the y value of $y = 5 + 3x$ and $y = 12 + 2x$ must be the same. The next step is to write one equation using x, then solve for x.

$$5 + 3x = 12 + 2x$$
$$5 + x = 12$$
$$x = 7$$

Use the value of x to find y. $y = 5 + 3(7) = 26$

The solution is (7, 26). This means that if you go on 7 rides, both plans will have the same cost of \$26.

Problems

Find the point of intersection (x, y) for each system of linear equations.

1. $y = x - 6$, $y = 12 - x$
2. $y = 3x - 5$, $y = x + 3$
3. $y = 2x + 16$, $y = 5x + 4$
4. $y = 3x - 5$, $y = 2x + 14$
5. $y = x + 7$, $y = 4x - 5$
6. $y = 7 - 3x$, $y = 2x - 8$

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Write a system of linear equations for each problem and use them to find a solution.

7. Jacques will wash the windows of a house for $15.00 plus $1.00 per window. Ray will wash them for $5.00 plus $2.00 per window. Let x be the number of windows and y be the total charge for washing them. Write an equation that represents how much each person charges to wash windows. Solve the system of equations and explain what the solution means and when it would be most economical to use each window washer.

8. Elle has moved to Hawksbluff for one year and wants to join a health club. She has narrowed her choices to two places: Thigh Hopes and ABSolutely fABulus. Thigh Hopes charges a fee of $95 to join and an additional $15 per month. ABSolutely fABulus charges a fee of $125 to join and a monthly fee of $12. Write two equations that represent each club's charges. What do your variables represent? Solve the system of equations and tell when the costs will be the same. Elle will only live there for one year, so which club will be less expensive?

9. Misha and Nora want to buy season passes for a ski lift but neither of them has the $225 needed to purchase a pass. Nora decides to get a job that pays $6.25 per hour. She has nothing saved right now but she can work four hours each week. Misha already has $80 and plans to save $15 of her weekly allowance. Who will be able to purchase a pass first?

10. Ginny is raising pumpkins to enter a contest to see who can grow the heaviest pumpkin. Her best pumpkin weighs 22 pounds and is growing at the rate of 2.5 pounds per week. Martha planted her pumpkins late. Her best pumpkin weighs 10 pounds but she expects it to grow 4 pounds per week. Assuming that their pumpkins grow at these rates, in how many weeks will their pumpkins weigh the same? How much will they weigh? If the contest ends in seven weeks, who will have the heavier pumpkin at that time?

11. Larry and his sister, Betty, are saving money to buy their own laptop computers. Larry has $215 and can save $35 each week. Betty has $380 and can save $20 each week. When will Larry and Betty have the same amount of money?

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Answers

1. (9, 3)
2. (4, 7)
3. (4, 24)
4. (19, 52)
5. (4, 11)
6. (3, –2)

7. Let $x =$ number of windows, $y =$ cost. Jacques: $y = 15 + 1x$; Roy: $y = 5 + 2x$. The solution is (10, 25), which means that the cost to wash 10 windows is $25. For fewer than 10 windows use Roy; for more than 10 windows, use Jacques.

8. Let $x =$ weeks, $y =$ total charges. Thigh Hopes: $y = 95 + 15x$; ABSolutely fABulus: $y = 125 + 12x$. The solution is (10, 245). At 10 months the cost at either club is $245. For 12 months use ABSolutely fABulus.

9. Let $x =$ weeks, $y =$ total savings. Misha: $y = 15x + 80$; Nora: $y = 25x$. The solution is (8, 200). Both of them will have $200 in 8 weeks, so Nora will have $225 in 9 weeks and be able to purchase the lift pass first. An alternative solution is to write both equations, then substitute 225 for *y* in each equation and solve for *x*. In this case, Nora can buy a ticket in 9 weeks, Misha in 9.67 weeks.

10. Let $x =$ weeks and $y =$ weight of the pumpkin. Ginny: $y = 2.5x + 22$; Martha: $y = 4x + 10$. The solution is (8, 42), so their pumpkins will weigh 42 pounds in 8 weeks. Ginny would win (39.5 pounds to 38 pounds for Martha).

11. Let $x =$ weeks, $y =$ total money saved. Larry: $y = 35x + 215$; Betty: $y = 20x + 380$. The solution is (11, 600). They will both have $600 in 11 weeks.

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RIGID TRANSFORMATIONS 6.1.1 – 6.1.4

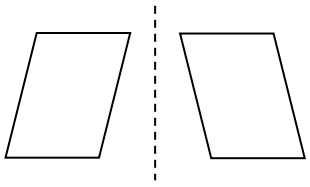
Studying transformations of geometric shapes builds a foundation for a key idea in geometry: congruence. In this introduction to transformations, the students explore three rigid motions: translations, reflections, and rotations. A translation slides a figure horizontally, vertically or both. A reflection flips a figure across a fixed line (for example, the *x*-axis). A rotation turns an object about a point (for example, (0, 0)). This exploration is done with simple tools that can be found at home (tracing paper) as well as with computer software. Students change the position and/or orientation of a shape by applying one or more of these motions to the original figure to create its image in a new position without changing its size or shape. Transformations also lead directly to studying symmetry in shapes. These ideas will help with describing and classifying geometric shapes later in the course.

For additional information, see the Math Notes box in Lesson 6.1.3 of the *Core Connections, Course 3* text.

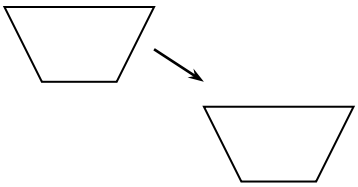
Example 1

Decide which transformation was used on each pair of shapes below. Some may be a combination of transformations.

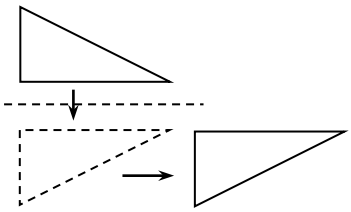
a.



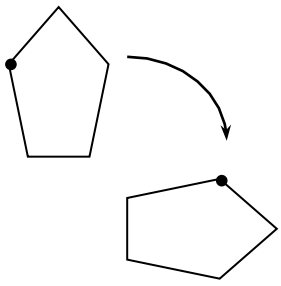
b.



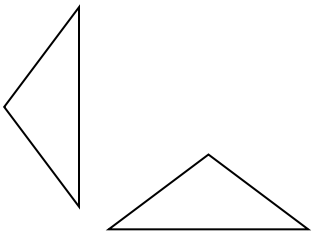
c.



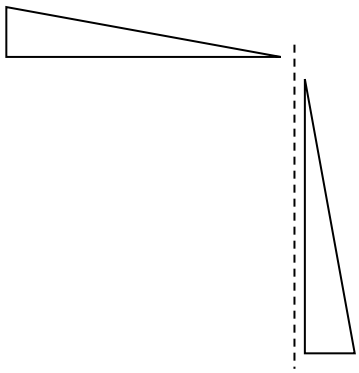
d.



e.



f.



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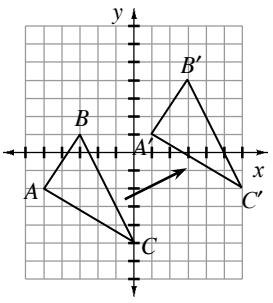
Identifying a single transformation is usually easy for students. In part (a), the parallelogram is reflected (flipped) across an invisible vertical line. (Imagine a mirror running vertically between the two figures. One figure would be the reflection of the other.) Reflecting a shape once changes its orientation, that is, how its parts "sit" on the flat surface. For example, in part (a), the two sides of the figure at left slant upwards to the right, whereas in its reflection at right, they slant upwards to the left. Likewise, the angles in the figure at left "switch positions" in the figure at right.

In part (b), the shape is translated (or slid) to the right and down. The orientation is the same. Part (c) shows a combination of transformations. First the triangle is reflected (flipped) across an invisible horizontal line. Then it is translated (slid) to the right. The pentagon in part (d) has been rotated (turned) clockwise to create the second figure. Imagine tracing the first figure on tracing paper, then holding the tracing paper with a pin at one point below the first pentagon, then turning the paper to the right (that is, clockwise) 90°. The second pentagon would be the result. Some students might see this as a reflection across a diagonal line. The pentagon itself could be, but with the added dot, the entire shape cannot be a reflection. If it had been reflected, the dot would have to be on the corner below the one shown in the rotated figure. The triangles in part (e) are rotations of each other (90° clockwise again). Part (f) shows another combination. The triangle is rotated (the horizontal side becomes vertical) but also reflected since the longest side of the triangle points in the opposite direction from the first figure.

Example 2

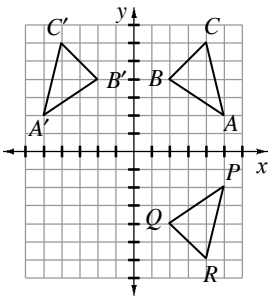
Translate (slide) ΔABC right six units and up three units. Give the coordinates of the new triangle.



The original vertices are $A(-5, -2)$, $B(-3, 1)$, and $C(0, -5)$. The new vertices are $A'(1, 1)$, $B'(3, 4)$, and $C'(6, -2)$. Notice that the change to each original point (x, y) can be represented by $(x + 6, y + 3)$.

Example 3

Reflect (flip) ΔABC with coordinates $A(5, 2)$, $B(2, 4)$, and $C(4, 6)$ across the y-axis to get $\Delta A'B'C'$. The key is that the reflection is the same distance from the y-axis as the original figure. The new points are $A'(-5, 2)$, $B'(-2, 4)$, and $C'(-4, 6)$. Notice that in reflecting across the y-axis, the change to each original point (x, y) can be represented by $(-x, y)$.



If you reflect ΔABC across the x-axis to get ΔPQR, then the new points are $P(5, -2)$, $Q(2, -4)$, and $R(4, -6)$. In this case, reflecting across the x-axis, the change to each original point (x, y) can be represented by $(x, -y)$.

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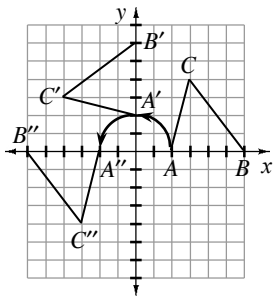
Example 4



Rotate (turn) ΔABC with coordinates $A(2, 0)$, $B(6, 0)$, and $C(3, 4)$ 90° counterclockwise about the origin $(0, 0)$ to get $\Delta A'B'C'$ with coordinates $A'(0, 2)$, $B'(0, 6)$, and $C'(-4, 3)$. Notice that for this 90° counterclockwise rotation about the origin, the change to each original point (x, y) can be represented by $(-y, x)$.

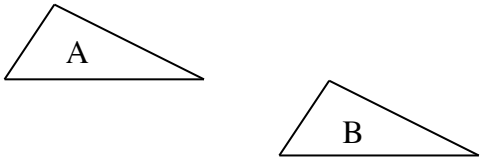
Rotating another 90° (180° from the starting location) yields $\Delta A''B''C''$ with coordinates $A''(-2, 0)$, $B''(-6, 0)$, and $C''(-3, -4)$.

For this 180° counterclockwise rotation about the origin, the change to each original point (x, y) can be represented by $(-x, -y)$. Similarly a 270° counterclockwise or 90° clockwise rotation about the origin takes each original point (x, y) to the point $(y, -x)$.

Problems

For each pair of triangles, describe the transformation that moves triangle A to the location of triangle B.

1.



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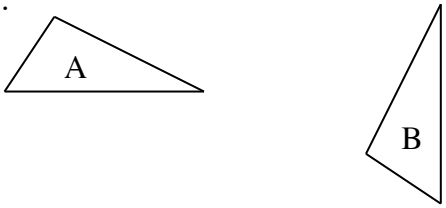

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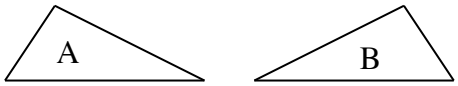
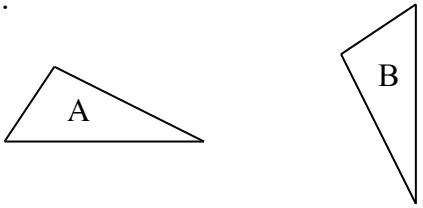


4.



For the following problems, refer to the figures below:

Figure A

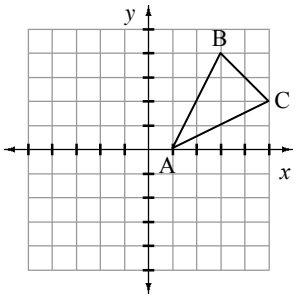


Figure B

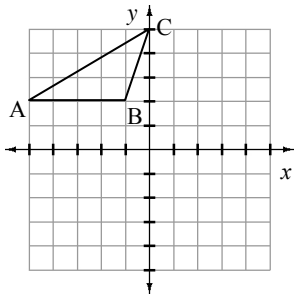
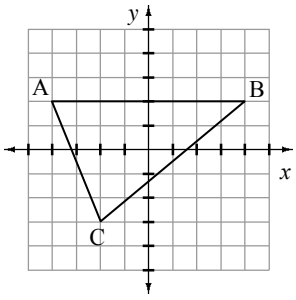


Figure C



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State the new coordinates after each transformation.

5. Slide figure A left 2 units and down 3 units.

6. Slide figure B right 3 units and down 5 units.

7. Slide figure C left 1 unit and up 2 units.

8. Flip figure A across the *x*-axis.

9. Flip figure B across the *x*-axis.

10. Flip figure C across the *x*-axis.

11. Flip figure A across the *y*-axis.

12. Flip figure B across the *y*-axis.

13. Flip figure C across the *y*-axis.

14. Rotate figure A 90º counterclockwise about the origin.

15. Rotate figure B 90º counterclockwise about the origin.

16. Rotate figure C 90º counterclockwise about the origin.

17. Rotate figure A 180º counterclockwise about the origin.

18. Rotate figure C 180º counterclockwise about the origin.

19. Rotate figure B 270º counterclockwise about the origin.

20. Rotate figure C 90º clockwise about the origin.

Answers (1 to 4 may vary; 5 to 20 given in the order A′, B′, C′)

1.	translation	2.	rotation and translation
3.	reflection	4.	rotation and translation
5.	(–1, –3), (1, 2), (3, –1)	6.	(–2, –3), (2, –3), (3, 0)
7.	(–5, 4), (3, 4), (–3, –1)	8.	(1, 0), (3, –4), (5, –2)
9.	(–5, –2), (–1, –2), (0, –5)	10.	(–4, –2), (4, –2), (–2, 3)
11.	(–1, 0), (–3, 4), (–5, 2)	12.	(5, 2), (1, 2), (0, 5)
13.	(4, 2), (–4, 2), (2, –3)	14.	(0, 1), (–4, 3), (–2, 5)
15.	(–2, –5), (–5, 0), (–2, –1)	16.	(–2, –4), (–2, 4), (3, –2)
17.	(–1, 0), (–3, –4), (–5, –2)	18.	(4, –2), (–4, –2), (2, 3)
19.	(2, 5), (2, 1), (5, 0)	20.	(2, 4), (2, –4), (–3, 2)

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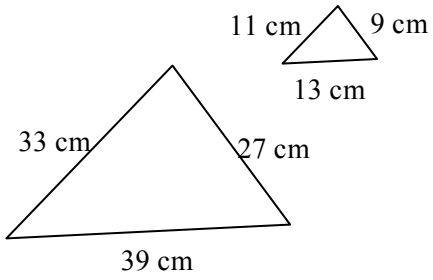
SIMILAR FIGURES 6.2.1 – 6.2.4

Two figures that have the same shape but not necessarily the same size are similar. In similar figures the measures of the corresponding angles are equal and the ratios of the corresponding sides are proportional. This ratio is called the scale factor. For information about corresponding sides and angles of similar figures see the Math Notes box in Lesson 6.2.2 of the *Core Connections, Course 3* text. For information about scale factor and similarity, see the Math Notes box in Lesson 6.2.6 of the *Core Connections, Course 3* text.

Example 1

Determine if the figures are similar. If so, what is the scale factor?

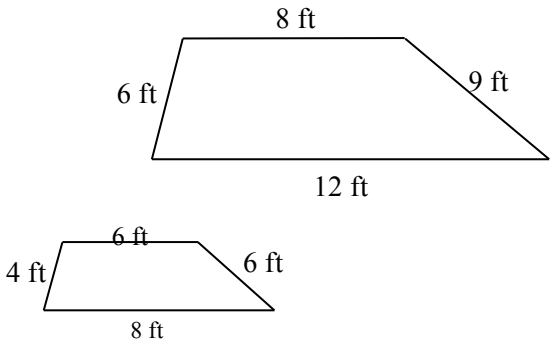


$\frac{39}{13} = \frac{33}{11} = \frac{27}{9} = \frac{3}{1}$ or 3

The ratios of corresponding sides are equal so the figures are similar. The scale factor that compares the small figure to the large one is 3 or 3 to 1. The scale factor that compares the large figure to the small figure is $\frac{1}{3}$ or 1 to 3.

Example 2

Determine if the figures are similar. If so, state the scale factor.



$\frac{6}{4} = \frac{12}{8} = \frac{9}{6}$ and all equal $\frac{3}{2}$.

$\frac{8}{6} = \frac{4}{3}$ so the shapes are not similar.

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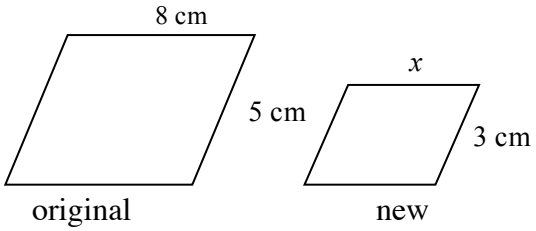
Example 3

Determine the scale factor for the pair of similar figures. Use the scale factor to find the side length labeled with a variable.



scale factor = $\frac{3}{5}$

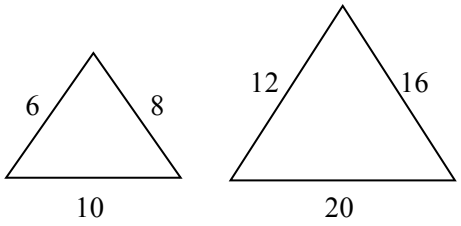
original $\cdot \frac{3}{5} \Rightarrow$ new

$8 \cdot \frac{3}{5} = x ; \Rightarrow x = \frac{24}{5} = 4.8$ cm

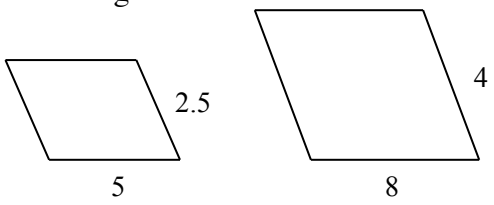
Problems

Determine if the figures are similar. If so, state the scale factor of the first to the second.

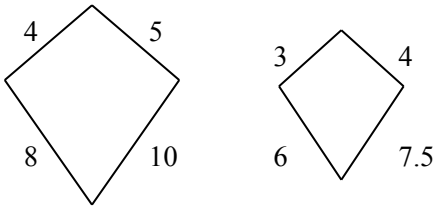
1.



2. Parallelograms

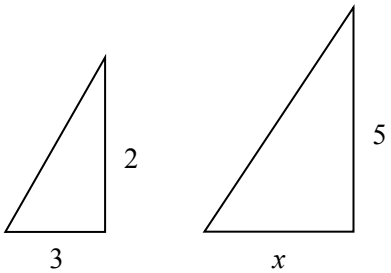


3. Kites

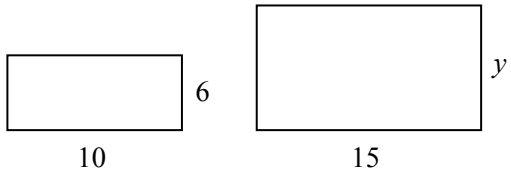


Determine the scale factor for each pair of similar figures. Use the scale factor to find the side labeled with the variable.

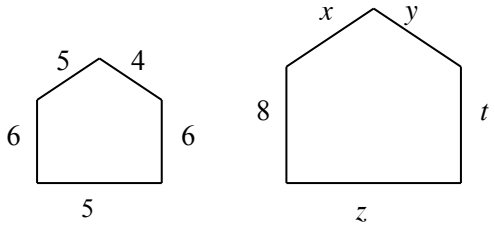
4.



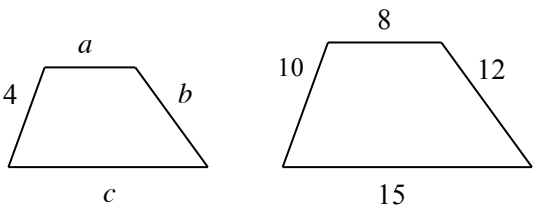
5.



6.



7.



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Answers

1. similar; 2

2. similar; $\frac{8}{5} = 1.6$

3. not similar

4. $\frac{5}{2}$; $x = 7.5$

5. $\frac{3}{2}$; $y = 9$

6. $\frac{4}{3}$; $x = \frac{20}{3} = 6\frac{2}{3}$, $y = \frac{16}{3} = 5\frac{1}{3}$, $t = 8$, $z = \frac{25}{3} = 8\frac{1}{3}$

7. $\frac{5}{2}$; $a = \frac{16}{5} = 3.2$, $b = \frac{24}{5} = 4.8$, $c = 6$

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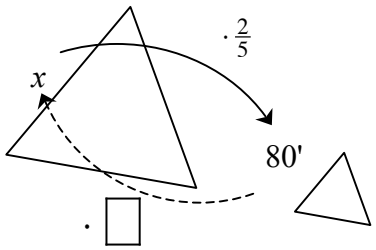
SCALING TO SOLVE PERCENT AND OTHER PROBLEMS 6.2.4 – 6.2.6

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient. Recall that a reciprocal is the multiplicative inverse of a number, that is, the product of the two numbers is 1. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, $\frac{1}{2}$ is $\frac{2}{1}$, and 5 is $\frac{1}{5}$.

Scaling may also be used with percentage problems where a quantity is increased or decreased by a certain percent. Scaling by a factor of 1 does not change the quantity. Increasing by a certain percent may be found by multiplying by (1 + the percent) and decreasing by a certain percent may be found by multiplying by (1 – the percent).

Example 1

The large triangle at right was reduced by a scale factor of $\frac{2}{5}$ to create a similar triangle. If the side labeled x now has a length of 80' in the new figure, what was the original length?



To undo the reduction, multiply 80' by the reciprocal of $\frac{2}{5}$, namely $\frac{5}{2}$, or divide 80' by $\frac{2}{5}$.

$80' \div \frac{2}{5}$ is the same as $80' \cdot \frac{5}{2}$, so $x = 200'$.

Example 2

Solve: $\frac{2}{3}x = 12$

Method 1: Use division and a Giant One

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3}x = 12$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(12)$$

$$x = 18$$

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Example 3

Samantha wants to leave a 15% tip on her lunch bill of $12.50. What scale factor should be used and how much money should she leave?

Since tipping increases the total, the scale factor is (1 + 15%) = 1.15.
She should leave (1.15)(12.50) = $14.38 or about $14.50.

Example 4

Carlos sees that all DVDs are on sales at 40% off. If the regular price of a DVD is $24.95, what is the scale factor and how much is the sale price?

If items are reduced 40%, the scale factor is (1 – 40%) = 0.60.
The sale price is (0.60)(24.95) = $14.97.

Problems

1. A rectangle was enlarged by a scale factor of $\frac{5}{2}$ and the new width is 40 cm. What was the original width?

2. A side of a triangle was reduced by a scale factor of $\frac{2}{3}$. If the new side is now 18 inches, what was the original side?

3. The scale factor used to create the design for a backyard is 2 inches for every 75 feet ($\frac{2}{75}$). If on the design, the fire pit is 0.5 inches away from the house, how far from the house, in feet, should the fire pit be dug?

4. After a very successful year, Cheap-Rentals raised salaries by a scale factor of $\frac{11}{10}$. If Luan now makes $14.30 per hour, what did she earn before?

5. Solve: $\frac{3}{4}x = 60$

6. Solve: $\frac{2}{5}x = 42$

7. Solve: $\frac{3}{5}y = 40$

8. Solve: $-\frac{8}{3}m = 6$

9. What is the total cost of a $39.50 family dinner after you add a 20% tip?

10. If the current cost to attend Magicland Park is now $29.50 per person, what will be the cost after a 8% increase?

11. Winter coats are on clearance at 60% off. If the regular price is $79, what is the sale price?

12. The company president has offered to reduce his salary 10% to cut expenses. If she now earns $175,000, what will be her new salary?

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Answers

1.	16 cm	2.	27 inches	3.	$18\frac{3}{4}$ feet	4.	\$13.00
5.	80	6.	105	7.	$66\frac{2}{3}$	8.	$-2\frac{1}{4}$
9.	\$47.40	10.	\$31.86	11.	\$31.60	12.	\$157,500

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CIRCLE GRAPHS 7.1.1

A circle graph (or pie chart) is a diagram that represents proportions of categorized data as parts of a circle. Each sector or wedge represents a percent or fraction of the circle. The fractions or percents must total 1, or 100%. Since there are 360º in a circle, the size of each sector (in degrees) is found by multiplying the fraction or percent by 360º.

For additional information, see the Math Notes box in Lesson 7.1.1 of the *Core Connections, Course 3* text.

Example 1

Ms. Sallee's class of 30 students was surveyed about the number of hours of homework done each night and here are the results:

less than 1 hour	3 students
1 to 2 hours	9 students
2 to 3 hours	12 students
3 to 4 hours	4 students
more than 4 hours	2 students

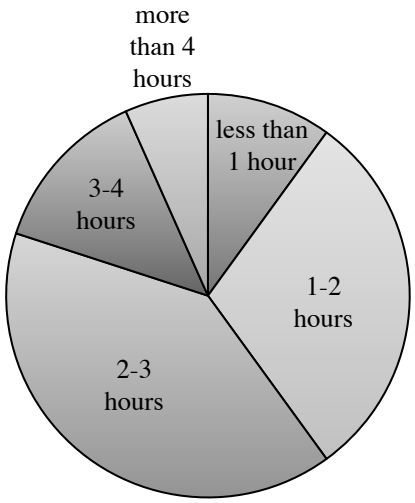
The proper size for the sectors is found as follows:

< 1: $\frac{3}{30} \cdot 360º = 36º$; $1–2$: $\frac{9}{30} \cdot 360º = 108º$

$2–3$: $\frac{12}{30} \cdot 360º = 144º$; $3–4$: $\frac{4}{30} \cdot 360º = 40º$

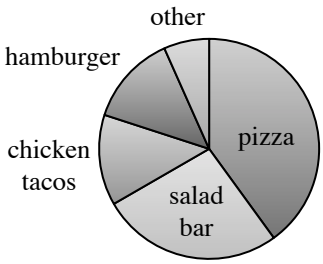
> 4: $\frac{2}{30} \cdot 360º = 20º$

The circle graph is shown at right.



Example 2

The 800 students at Central Middle School were surveyed to determine their favorite school lunch item. The results are shown below.



Use the circle graph at left to answer each question.

a. Which lunch item was most popular?

b. Approximately how many students voted for the salad bar?

c. Which two lunch items appear to have equal popularity?

Answers: a. pizza is the largest sector; b. $\frac{1}{4} \cdot 800 = 200$;
c. hamburger and chicken tacos have the same size sectors

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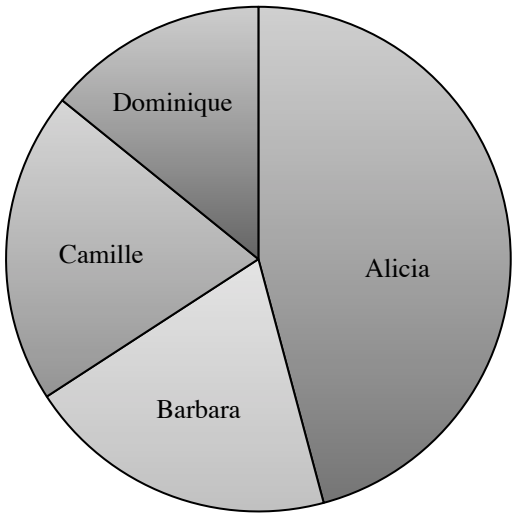
Problems

For problems 1 through 3 use the circle graph at right. The graph shows the results of the 1200 votes for prom queen.



1. Who won the election?

2. Did the person who won the election get more than half of the votes?

3. Approximately how many votes did Camille receive?

4. Of the milk consumed in the United States, 30% is whole, 50% is low fat, and 20% is skim. Draw a circle graph to show this data.

5. On an average weekday, Sam's time is spent as follows: sleep 8 hours, school 6 hours, entertainment 2 hours, homework 3 hours, meals 1 hour, and job 4 hours. Draw a circle graph to show this data.

6. Records from a pizza parlor show the most popular one-item pizzas are: pepperoni 42%, sausage 25%, mushroom 10%, olive 9% and the rest were others. Draw a circle graph to show this data.

7. To pay for a 200 billion dollar state budget, the following monies were collected: income taxes 90 billion dollars, sales taxes 74 billion dollars, business taxes 20 billion dollars, and the rest were from miscellaneous sources. Draw a circle graph to show this data.

8. Greece was the host country for the 2004 Summer Olympics. The Greek medal count was 6 gold, 6 silver, and 4 bronze. Draw a circle graph to show this data.

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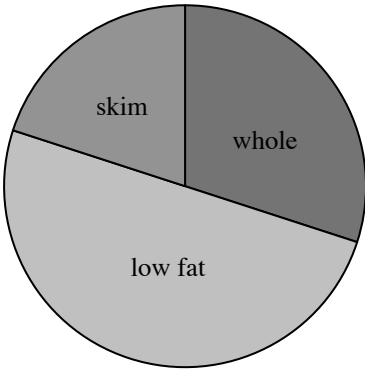
Answers

1. Alicia

2. No, Alicia's sector is less than half of a circle.

3. Approximately 240 votes.

4.



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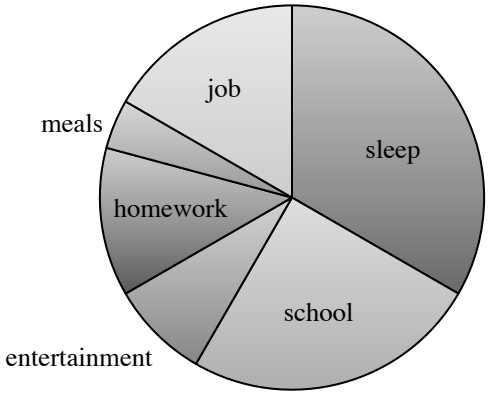

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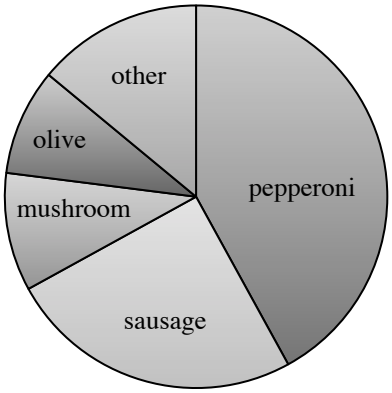
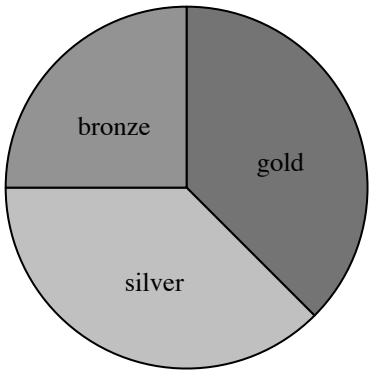


7.

8.



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SCATTERPLOTS, ASSOCIATION, AND LINE OF BEST FIT 7.1.2 – 7.1.3

Data that is collected by measuring or observing naturally varies. A scatterplot helps students decide is there is a relationship (an **association**) between two numerical variables.

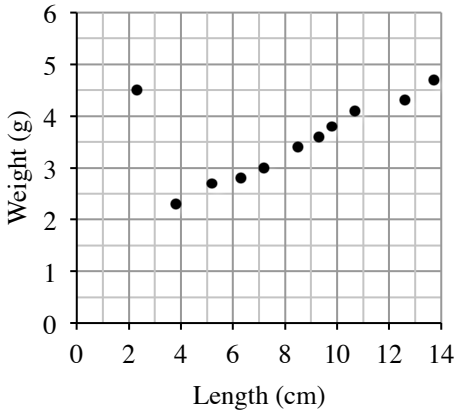
If there is a possible linear relationship, the trend can be shown graphically with a line of best fit on the scatterplot. In this course, students use a ruler to estimate a line of best fit. The equation of the best-fit line can be determined from the slope and the *y*-intercept.

An association is often described by its form, direction, strength, and outliers. See the Math Notes boxes in Lessons 7.1.2, 7.1.3, and 7.3.2 of the *Core Connections, Course 3* text.

For additional examples and practice, see the *Core Connections, Course 3* Checkpoint 9 materials.

Example 1

Sam collected data by measuring the pencils of her classmates. She recorded the length of the painted part of each pencil and its weight. Her data is shown on the graph at right.



a. Describe the association between weight and length of the painted part of the pencil.

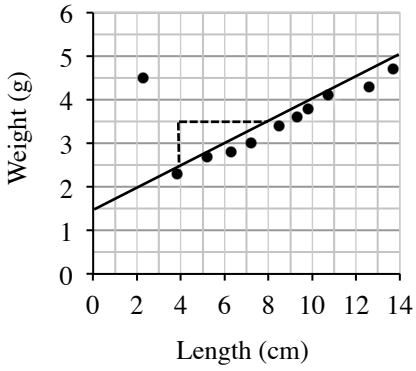
b. Create a line of best fit where *y* is the weight of the pencil in grams and *x* is the length of the painted part of the pencil in centimeters.

c. Sam's teacher has a pencil where the painted part is 11.5 cm long. Predict the weight of the teacher's pencil using the equation found in part (b).

Answer:

a. There is a strong positive linear association with one apparent outlier at 2.3 cm.

b. The equation of the line of best fit is approximately $y = \frac{1}{4}x + 1.5$. See graph at right.



c. $\frac{1}{4}(11.5) + 1.5 \approx 4.4$ g

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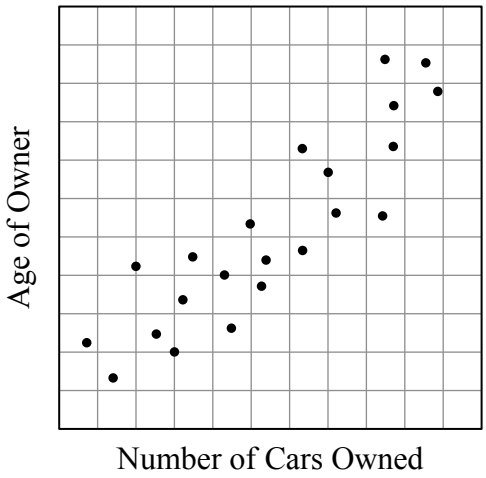
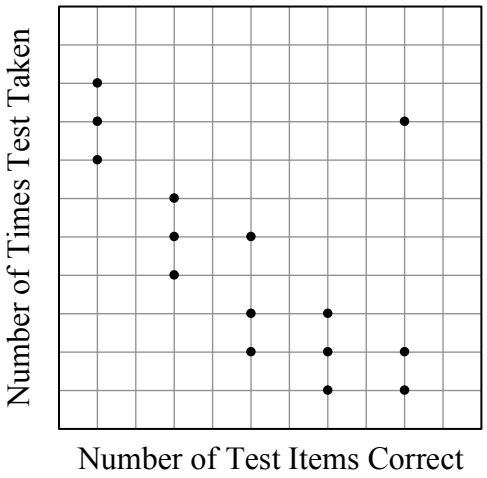
Problems

In problems 1 through 4 describe (if they exist), the form, direction, strength, and outliers of the scatterplot.

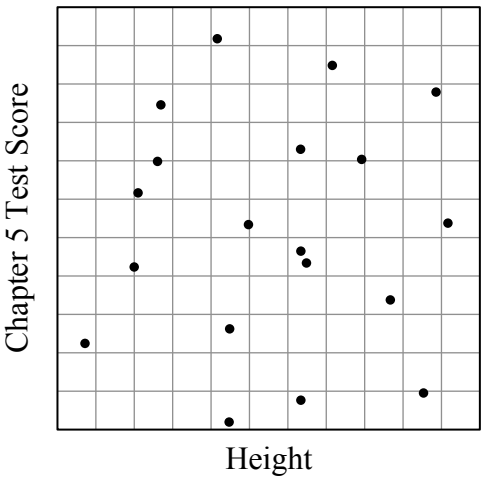
1.



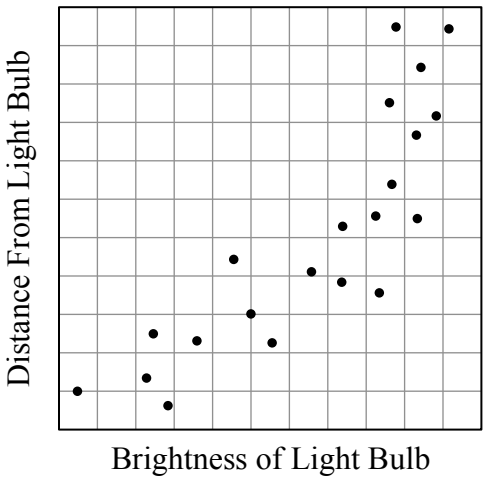
2.



3.



4.



5. Dry ice (frozen carbon dioxide) has the unusual property of sublimating (changing directly from a solid to a gas). Giulia's father uses dry ice to keep the glasses in the restaurant cold. Since dry ice sublimates in the restaurant cooler, Giulia was curious how long a piece of dry ice would last. She collected the data shown in the table at right.

   Draw a scatterplot and a line of best fit. What is the approximate equation of the line of best fit?

# of hours after noon	Weight of dry ice (oz)
0	15.3
1	14.7
2	14.3
3	13.6
4	13.1
5	12.5
6	11.9
7	11.5
8	11.0
9	10.6
10	10.2

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6. Ranger Scott is responsible for monitoring the population of the elusive robins in McNeil State Park. He would like to find a relationship between the elm trees (their preferred nesting site) and the number of robins in the park. He randomly selects 7 different areas in the park and painstakingly counts the elms and robins in each area.

Elms	8	13	4	5	10	9	4
Robins	5	9	3	5	7	7	5

a. Make a scatterplot on graph paper and describe the association.

b. Sketch the line of best fit on your scatterplot. Find the equation of the line of best fit.

c. Based on the equation, how many robins should Ranger Scott expect to find in an area with 6 elm trees?

7. A study was done for a vitamin supplement that claims to shorten the length of the common cold. The data the scientists collected from ten patients in an early study are shown in the table below.

Number of months taking supplement	0.5	2.5	1	2	0.5	1	2	1	1.5	2.5
Number of days cold lasted	4.5	1.6	3	1.8	5	4.2	2.4	3.6	3.3	1.4

a. Create a scatterplot and describe the association.

b. Model the data with a line of best fit. Use x to represent the number of months taking the supplement and y to represent the length of the cold.

c. According to your model, how many days do you expect a cold to last for patient taking the supplement for 1.5 months?

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Answers

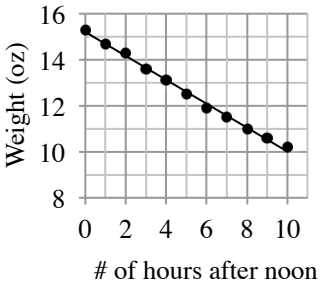
1. Moderate, positive, linear association with no outliers.

2. Strong, negative, linear association with an outlier.

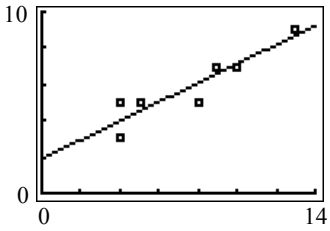
3. No association.

4. Strong, positive, curved association.

5. $y = -\frac{1}{2}x + 15.3$



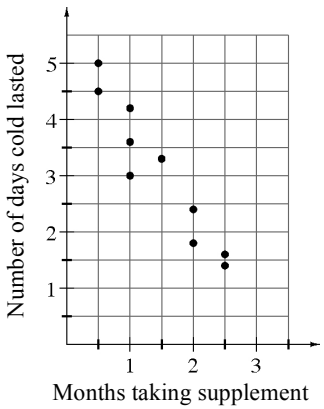
6. Strong, positive linear association with no outliers.
   $y = \frac{1}{2}x + 2$
   5 robins



7. a. The form is linear, the direction is negative, the strength is moderate, and there are no apparent outliers.

   b. $y = -\frac{5}{3}x + 5$

   c. $-\frac{5}{3}(\frac{3}{2}) + 5 = 2\frac{1}{2}$ days



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The slope of a line is the ratio of the change in y to the change in x between any two points on a line. Slope indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right.

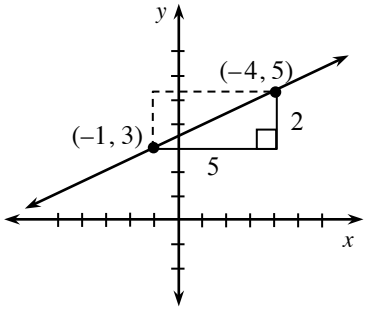
Slope is determined by the ratio $\frac{\text{vertical change}}{\text{horizontal change}}$ between *any* two points on a line.

For lines that go up (from left to right), the sign of the slope is positive (the change in y is positive). For lines that go down (left to right), the sign of the slope is negative (the change is y is negative). A horizontal line has zero slope while the slope of a vertical line is undefined.

For additional information see the Math Notes box in Lesson 7.2.4 of the *Core Connections, Course 3* text.

Example 1

Write the slope of the line containing the points (–1, 3) and (4, 5).



First graph the two points and draw the line through them.

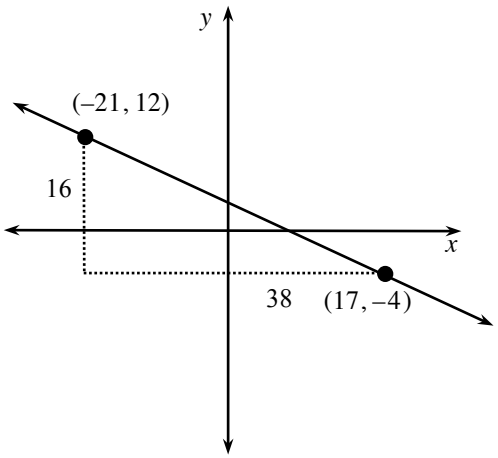
Look for and draw a slope triangle using the two given points.

Write the ratio $\frac{\text{vertical change in } y}{\text{horizontal change in } x}$ using the legs of the right triangle: $\frac{2}{5}$.

Assign a positive or negative value to the slope (this one is positive) depending on whether the line goes up (+) or down (–) from left to right.

Example 2

If the points are inconvenient to graph, use a "generic slope triangle," visualizing where the points lie with respect to each other. For example, to find the slope of the line that contains the points (–21, 12) and (17, –4), sketch the graph at right to approximate the position of the two points, draw a slope triangle, find the length of the leg of each triangle, and write the ratio $\frac{y}{x} = \frac{16}{38}$, then simplify. The slope is $-\frac{8}{19}$ since the change in y is negative (decreasing).



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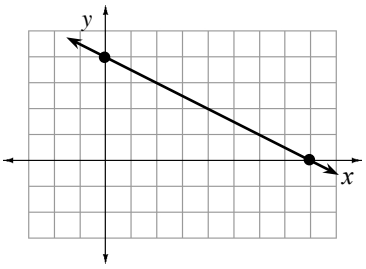
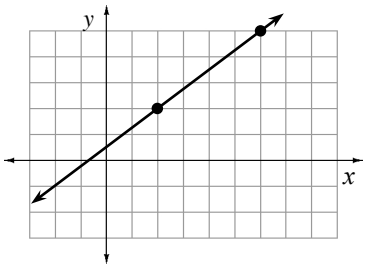
Problems

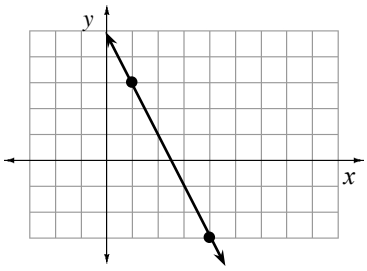
Write the slope of the line containing each pair of points.

1. (3, 4) and (5, 7)
2. (5, 2) and (9, 4)
3. (1, –3) and (–4, 7)
4. (–2, 1) and (2, –2)
5. (–2, 3) and (4, 3)
6. (32, 12) and (12, 20)

Determine the slope of each line using the *highlighted points*.

7. 

8. 

9. 

Answers

1. $\frac{3}{2}$
2. $\frac{1}{2}$
3. –2
4. $-\frac{3}{4}$
5. 0
6. $-\frac{2}{5}$
7. $-\frac{1}{2}$
8. $\frac{3}{4}$
9. –2

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SIMPLE AND COMPOUND INTEREST 8.1.1 – 8.1.3

In Course 2 students are introduced to simple interest, the interest is paid only on the original amount invested. The formula for simple interest is: $I = Prt$ and the total amount including interest would be: $A = P + I$. In Course 3, students are introduced to compound interest using the formula: $A = P(1 + r)^n$. Compound interest is paid on both the original amount invested and the interest previously earned. Note that in these formulas, P = principal (amount invested), r = rate of interest, t and n both represent the number of time periods for which the total amount A, is calculated and I = interest earned.

For additional information, see the Math Notes box in Lesson 8.1.3 of the *Core Connections, Course 3* text.

Example 1

Wayne earns 5.3% simple interest for 5 years on $3000. How much interest does he earn and what is the total amount in the account?

Put the numbers in the formula $I = Prt$. $I = 3000(5.3\%)5$

Change the percent to a decimal. $= 3000(0.053)5$

Multiply. $= 795$ Wayne would earn $795 interest.

Add principal and interest. $3000 + $795 = $3795 in the account

Example 2

Use the numbers in Example 1 to find how much money Wayne would have if he earned 5.3% interest compounded annually.

Put the numbers in the formula $A = P(1 + r)^n$. $A = 3000(1 + 5.3\%)^5$

Change the percent to a decimal. $= 3000(1 + 0.053)^5$ or $3000(1.053)^5$

Multiply. $= 3883.86$

Wayne would have $3883.86.

Students are asked to compare the difference in earnings when an amount is earning simple or compound interest. In these examples, Wayne would have $88.86 more with compound interest than he would have with simple interest: $3883.86 – $3795 = $88.86.

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Problems

Solve the following problems.

1. Tong loaned Jody $50 for a month. He charged 5% simple interest for the month. How much did Jody have to pay Tong?

2. Jessica's grandparents gave her $2000 for college to put in a savings account until she starts college in four years. Her grandparents agreed to pay her an additional 7.5% simple interest on the $2000 for every year. How much extra money will her grandparents give her at the end of four years?

3. David read an ad offering $8\frac{3}{4}\%$ simple interest on accounts over $500 left for a minimum of 5 years. He has $500 and thinks this sounds like a great deal. How much money will he earn in the 5 years?

4. Javier's parents set an amount of money aside when he was born. They earned 4.5% simple interest on that money each year. When Javier was 15, the account had a total of $1012.50 interest paid on it. How much did Javier's parents set aside when he was born?

5. Kristina received $125 for her birthday. Her parents offered to pay her 3.5% simple interest per year if she would save it for at least one year. How much interest could Kristina earn?

6. Kristina decided she would do better if she put her money in the bank, which paid 2.8% interest compounded annually. Was she right?

7. Suppose Jessica (from problem 2) had put her $2000 in the bank at 3.25% interest compounded annually. How much money would she have earned there at the end of 4 years?

8. Mai put $4250 in the bank at 4.4% interest compounded annually. How much was in her account after 7 years?

9. What is the difference in the amount of money in the bank after five years if $2500 is invested at 3.2% interest compounded annually or at 2.9% interest compounded annually?

10. Ronna was listening to her parents talking about what a good deal compounded interest was for a retirement account. She wondered how much money she would have if she invested $2000 at age 20 at 2.8% annual interest compounded quarterly (four times each year) and left it until she reached age 65. Determine what the value of the $2000 would become.

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Answers

1. $I = 50(0.05)1 = \$2.50$; Jody paid back \$52.50.

2. $I = 2000(0.075)4 = \$600$

3. $I = \$500(0.0875)5 = \$218.75$

4. $\$1012.50 = x(0.045)15$; $x = \$1500$

5. $I = 125(0.035)1 = \$4.38$

6. $A = 125(1 + 0.028)^1 = \$128.50$; No, for one year she needs to take the higher interest rate if the compounding is done annually. Only after one year will compounding earn more than simple interest.

7. $A = 2000(1 + 0.0325)^4 = \$2272.95$

8. $A = 4250(1 + 0.044)^7 = \$5745.03$

9. $A = 2500(1 + 0.032)^5 – 2500(1 + 0.029)^5 = \$2926.43 – \$2884.14 = \$42.29$

10. $A = 2000(1 + 0.007)^{180}$ (because $45 \cdot 4 = 180$ quarters) $= \$7019.96$

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EXPONENTS AND SCIENTIFIC NOTATION 8.2.1 – 8.2.4

EXPONENTS

In the expression 5^2, 5 is the base and 2 is the exponent. For x^a, x is the base and a is the exponent. 5^2 means $5 \cdot 5$ and 5^3 means $5 \cdot 5 \cdot 5$, so you can write $\frac{5^5}{5^2}$ (which means $5^5 \div 5^2$) or you can write it like this: $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5}$.

You can use the Giant One to find the numbers in common. There are two Giant Ones, namely, $\frac{5}{5}$ twice, so $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^3$ or 125. Writing 5^3 is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^7}{x^3}$ means $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$. The Giant One here is $\frac{x}{x}$ (three of them). The answers is x^4.

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$, which is 5^5. $(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$, which is 5^6.

When the problems have variables such as $x^4 \cdot x^5$, you only need to add the exponents. The answer is x^9. If the problem is $(x^4)^5$ (x^4 to the fifth power) it means $x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4$. The answer is x^{20}. You multiply exponents in this case.

If the problem is $\frac{x^{10}}{x^4}$, you subtract the bottom exponent from the top exponent (10 – 4).

The answer is x^6. You can also have problems like $\frac{x^{10}}{x^{-4}}$. You still subtract, 10 – (–4) is 14, and the answer is x^{14}.

You need to be sure the bases are the same to use these laws. $x^5 \cdot y^6$ cannot be further simplified.

In general the laws of exponents are:

$$x^a \cdot x^b = x^{(a+b)} \qquad (x^a)^b = x^{ab} \qquad \frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1 \qquad x^{-n} = \frac{1}{x^n} \qquad (x^a y^b)^c = x^{ac} y^{bc}$$

These rules hold if $x \neq 0$ and $y \neq 0$.

For additional information, see Math Notes box in Lesson 8.2.4 of the *Core Connections, Course 3* text.

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Examples

a. $x^8 \cdot x^7 = x^{15}$

b. $\frac{x^{19}}{x^{13}} = x^6$

c. $(z^8)^3 = z^{24}$

d. $(x^2y^3)^4 = x^8y^{12}$

e. $\frac{x^4}{x^{-3}} = x^7$

f. $(2x^2y^3)^2 = 4x^4y^6$

g. $(3x^2y^{-2})^3 = 27x^6y^{-6}$ or $\frac{27x^6}{y^6}$

h. $\frac{x^8y^5z^2}{x^3y^6z^{-2}} = \frac{x^5z^4}{y}$ or $x^5y^{-1}z^4$

i. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

j. $5^2 \cdot 5^{-4} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Problems

Simplify each expression.

1. $5^2 \cdot 5^4$
2. $x^3 \cdot x^4$
3. $\frac{5^{16}}{5^{14}}$
4. $\frac{x^{10}}{x^6}$
5. $(5^3)^3$
6. $(x^4)^3$
7. $(4x^2\,y^3)^4$
8. $\frac{5^2}{5^{-3}}$
9. $5^5 \cdot 5^{-2}$
10. $(y^2)^{-3}$
11. $(4a^2b^{-2})^3$
12. $\frac{x^5y^4z^2}{x^4y^3z^2}$
13. $\frac{x^6y^2z^3}{x^{-2}y^3z^{-1}}$
14. $4x^2 \cdot 2x^3$
15. 4^{-2}
16. 3^{-3}
17. $6^3 \cdot 6^{-2}$
18. $(3^{-1})^2$

Answers

1. 5^6
2. x^7
3. 5^2
4. x^4
5. 5^9
6. x^{12}
7. $256x^8y^{12}$
8. 5^5
9. 5^3
10. y^{-6} or $\frac{1}{y^6}$
11. $64a^6b^{-6}$ or $\frac{64a^6}{b^6}$
12. xy
13. $\frac{x^8z^4}{y}$ or $x^8y^{-1}z^4$
14. $8x^5$
15. $\frac{1}{16}$
16. $\frac{1}{27}$
17. 6
18. $\frac{1}{9}$

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SCIENTIFIC NOTATION

Scientific notation is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as the product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent (power of 10).
- The factors are separated by a multiplication sign.
- A positive exponent indicates a number whose absolute value is greater than one.
- A negative exponent indicates a number whose absolute value is less than one.

Scientific Notation	Standard Form
5.32×10^{11}	532,000,000,000
2.61×10^{-15}	0.00000000000000261

It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number 5.32×10^{11} means $5.32 \times 100{,}000{,}000{,}000$. Thus, two of the 11 places in the standard form of the number are the 3 and the 2 in 5.32. Standard form in this case is 532,000,000,000. In this example you are moving the decimal point to the right 11 places to find standard form.

The number 2.61×10^{-15} means $2.61 \times 0.000000000000001$.
You are moving the decimal point to the left 15 places to find standard form.
Here the standard form is 0.00000000000000261.

For additional information, see the Math Notes box in Lesson 8.2.3 of the *Core Connections, Course 3* text.

Example 1

Write each number in standard form.

$7.84 \times 10^{8} \Rightarrow 784{,}000{,}000$ and $3.72 \times 10^{-3} \Rightarrow 0.00372$

When taking a number in standard form and writing it in scientific notation, remember there is only *one* digit before the decimal point, that is, the number must be between 1 and 9, inclusive.

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Example 2

$52{,}050{,}000 \Rightarrow 5.205\times10^{7}$ and $0.000372 \Rightarrow 3.72\times10^{-4}$

The exponent denotes the number of places you move the decimal point in the standard form. In the first example above, the decimal point is at the end of the number and it was moved 7 places. In the second example above, the exponent is negative because the original number is very small, that is, less than one.

Problems

Write each number in standard form.

1. 7.85×10^{11} 2. 1.235×10^{9} 3. 1.2305×10^{3} 4. 3.89×10^{-7} 5. 5.28×10^{-4}

Write each number in scientific notation.

6. 391,000,000,000 7. 0.0000842 8. 123056.7 9. 0.000000502

10. 25.7 11. 0.035 12. 5,600,000 13. 1346.8

14. 0.000000000006 15. 634,700,000,000,000

Note: On your scientific calculator, displays like 4.357^{12} and 3.65^{-3} are numbers expressed in scientific notation. The first number means 4.357×10^{12} and the second means 3.65×10^{-3}. The calculator does this because there is not enough room on its display window to show the entire number.

Answers

1. 785,000,000,000 2. 1,235,000,000 3. 1230.5

4. 0.000000389 5. 0.000528 6. 3.91×10^{11}

7. 8.42×10^{-5} 8. 1.230567×10^{5} 9. 5.02×10^{-7}

10. 2.57×10^{1} 11. 3.5×10^{-2} 12. 5.6×10^{6}

13. 1.3468×10^{3} 14. 6.0×10^{-12} 15. 6.347×10^{14}

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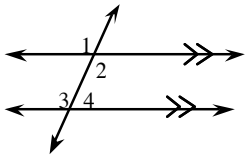
PROPERTIES OF ANGLES, LINES, AND TRIANGLES 9.1.1 – 9.1.4

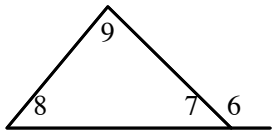
Students learn the relationships created when two parallel lines are intersected by a transversal. They also study angle relationships in triangles.

Parallel lines



- corresponding angles are equal: $m\angle 1 = m\angle 3$
- alternate interior angles are equal: $m\angle 2 = m\angle 3$
- $m\angle 2 + m\angle 4 = 180^\circ$

Triangles

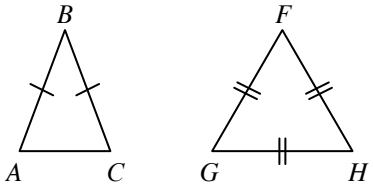


- $m\angle 7 + m\angle 8 + m\angle 9 = 180^\circ$
- $m\angle 6 = m\angle 8 + m\angle 9$ (exterior angle = sum remote interior angles)

Also shown in the above figures:

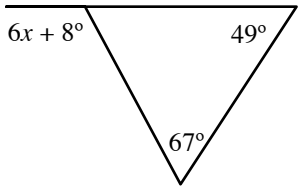
- vertical angles are equal: $m\angle 1 = m\angle 2$
- linear pairs are supplementary: $m\angle 3 + m\angle 4 = 180^\circ$ and $m\angle 6 + m\angle 7 = 180^\circ$

In addition, an isosceles triangle, ΔABC, has $\overline{BA} = \overline{BC}$ and $m\angle A = m\angle C$. An equilateral triangle, ΔGFH, has $\overline{GF} = \overline{FH} = \overline{HG}$ and $m\angle G = m\angle F = m\angle H = 60^\circ$.



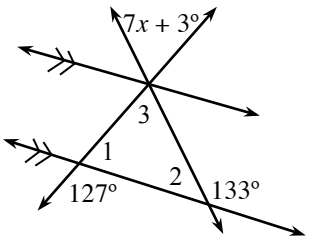
For more information, see the Math Notes boxes in Lessons 9.1.2, 9.1.3, and 9.1.4 of the *Core Connections, Course 3* text.

Example 1



Solve for x.
Use the Exterior Angle Theorem: $6x + 8^\circ = 49^\circ + 67^\circ$
$6x^\circ = 108^\circ \Rightarrow x = \frac{108^\circ}{6} \Rightarrow x = 18^\circ$

Example 2



Solve for x.
There are a number of relationships in this diagram. First, $\angle 1$ and the 127° angle are supplementary, so we know that $m\angle 1 + 127^\circ = 180^\circ$ so $m\angle 1 = 53^\circ$. Using the same idea, $m\angle 2 = 47^\circ$. Next, $m\angle 3 + 53^\circ + 47^\circ = 180^\circ$, so $m\angle 3 = 80^\circ$. Because angle 3 forms a vertical pair with the angle marked $7x + 3^\circ$, $80^\circ = 7x + 3^\circ$, so $x = 11^\circ$.

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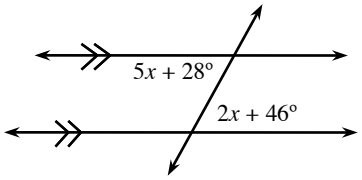
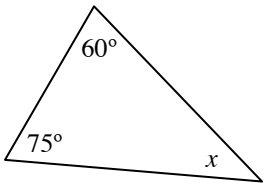
Example 3

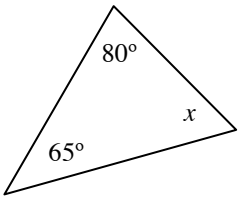


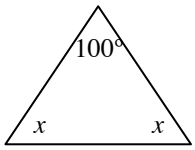
Find the measure of the acute alternate interior angles. Parallel lines mean that alternate interior angles are equal, so $5x + 28° = 2x + 46° \Rightarrow 3x = 18° \Rightarrow x = 6°$. Use either algebraic angle measure: $2(6°) + 46° = 58°$ for the measure of the acute angle.

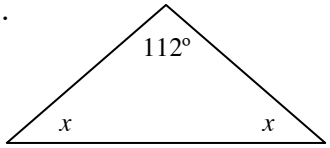
Problems

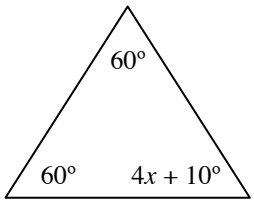
Use the geometric properties you have learned to solve for x in each diagram and write the property you use in each case.

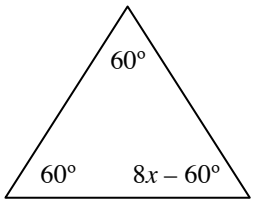
1. 

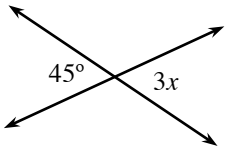
2. 

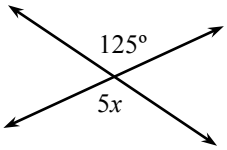
3. 

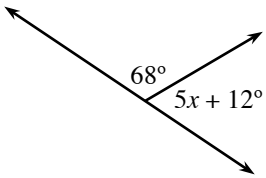
4. 

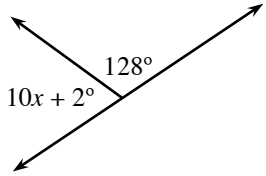
5. 

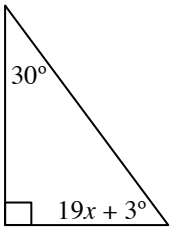
6. 

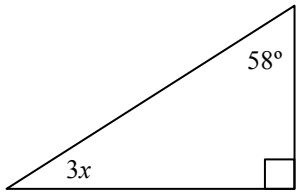
7. 

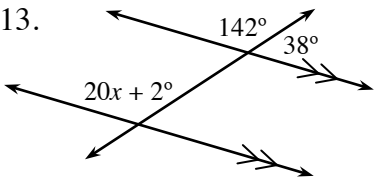
8. 

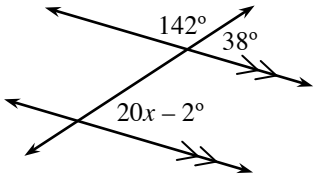
9. 

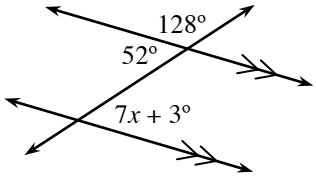
10. 

11. 

12. 

13. 

14. 

15. 

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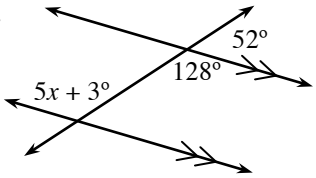
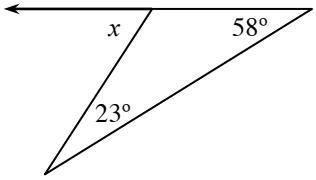
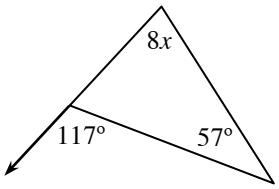
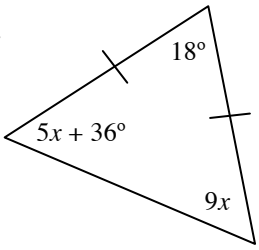
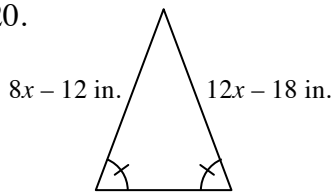
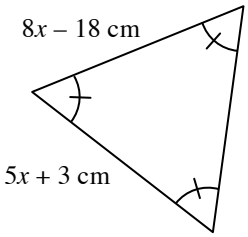
16. 

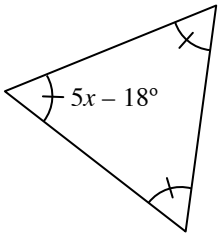
17. 

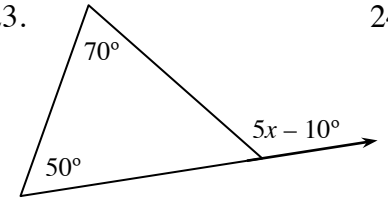
18. 

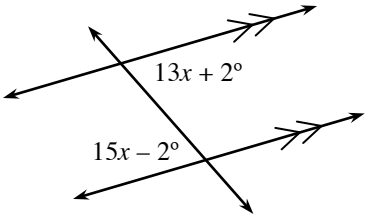
19. 

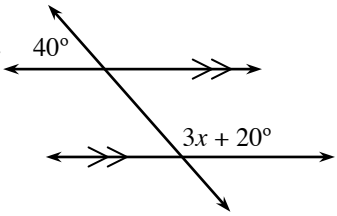
20. 

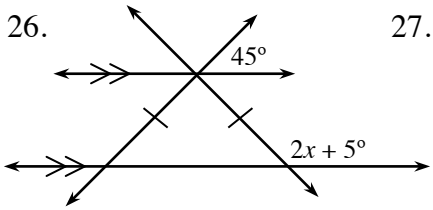
21. 

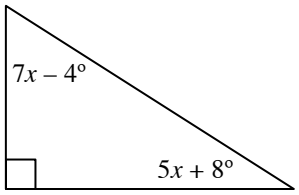
22. 

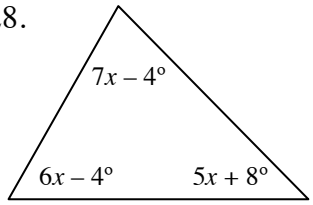
23. 

24. 

25. 

26. 

27. 

28. 

Answers

1.	45°	2.	35°	3.	40°	4.	34°	5.	12.5°	6.	15°
7.	15°	8.	25°	9.	20°	10.	5°	11.	3°	12.	$10\frac{2}{3}$°
13.	7°	14.	2°	15.	7°	16.	25°	17.	81°	18.	7.5°
19.	9°	20.	7.5°	21.	7°	22.	15.6°	23.	26°	24.	2°
25.	40°	26.	65°	27.	$7\frac{1}{6}$°	28.	10°				

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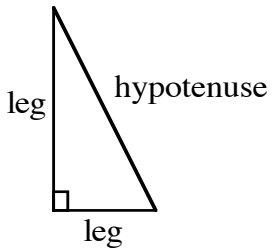
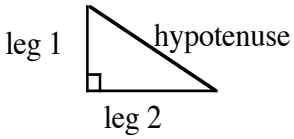
A right triangle is a triangle in which the two shorter sides form a right angle. The shorter sides are called legs. Opposite the right angle is the third and longest side called the hypotenuse.



The Pythagorean Theorem states that for any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$(\text{leg } 1)^2 + (\text{leg } 2)^2 = (\text{hypotenuse})^2$

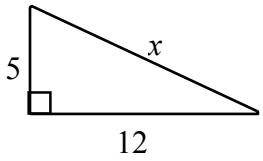


For additional information, see Math Notes box in Lesson 9.2.3 of the *Core Connections, Course 3* text.

Example 1

Use the Pythagorean Theorem to find x.

a.



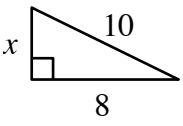
$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$169 = x^2$

$13 = x$

b.



$x^2 + 8^2 = 10^2$

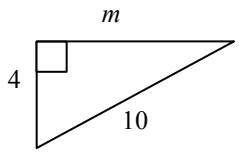
$x^2 + 64 = 100$

$x^2 = 36$

$x = 6$

Example 2

Not all problems will have exact answers. Use square root notation and your calculator.



$4^2 + m^2 = 10^2$

$16 + m^2 = 100$

$m^2 = 84$

$m = \sqrt{84} \approx 9.17$

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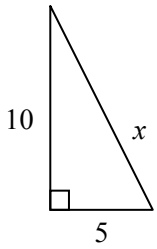
Example 3

A guy wire is needed to support a tower. The wire is attached to the ground five meters from the base of the tower. How long is the wire if the tower is 10 meters tall?

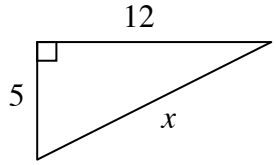
First draw a diagram to model the problem, then write an equation using the Pythagorean Theorem and solve it.

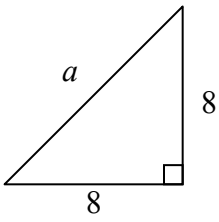


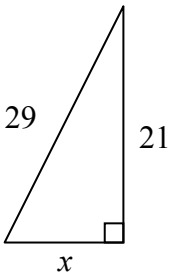
$$x^2 = 10^2 + 5^2$$
$$x^2 = 100 + 25$$
$$x^2 = 125$$
$$x = \sqrt{125} \approx 11.18 \text{ cm}$$

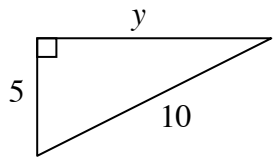
Problems

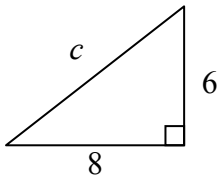
Write an equation and solve it to find the length of the unknown side. Round answers to the nearest hundredth.

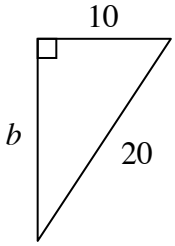
1. 

2. 

3. 

4. 

5. 

6. 

Draw a diagram, write an equation, and solve it. Round answers to nearest hundredth.

7. Find the diagonal of a television screen 30 inches wide by 35 inches tall.

8. A 9-meter ladder is one meter from the base of a building. How high up the building will the ladder reach?

9. Sam drove eight miles south and then five miles west. How far is he from his starting point?

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10. The length of the hypotenuse of a right triangle is six centimeters. If one leg is four centimeters, how long is the other leg?

11. Find the length of a path that runs diagonally across a 55-yard by 100-yard field.

12. How long an umbrella will fit in the bottom of a suitcase 1.5 feet by 2.5 feet?

Answers

1.	13	2.	11.31	3.	20	4.	8.66	5.	10	6.	17.32
7.	46.10 in.	8.	8.94 m	9.	9.43 mi	10.	4.47 cm	11.	114.13 yd	12.	2.92 ft

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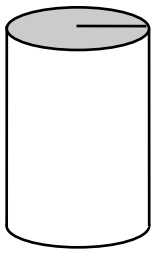
VOLUME OF A CYLINDER



The volume of a cylinder is the area of its base multiplied by its height:

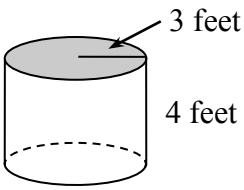
$$V = B \cdot h$$

Since the base of a cylinder is a circle of area $A = r^2\pi$, we can write:

$$V = r^2\pi h$$

For additional information, see the Math Notes box in Lesson 10.1.2 of the *Core Connections, Course 3* text.

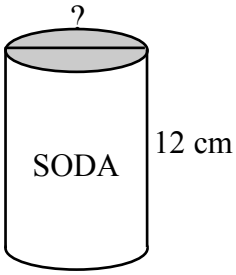
Example 1



Find the volume of the cylinder above. Use a calculator for the value of π.

Volume $= r^2\pi h$
$= (3)^2\pi\,(4)$
$= 36\pi$
$= 113.10 \text{ ft}^3$

Example 2



The soda can above has a volume of 355 cm^3 and a height of 12 cm. What is its diameter? Use a calculator for the value of π.

Volume $= r^2\pi h$
$355 = r^2\pi(12)$
$\frac{355}{12\pi} = r^2$
$9.42 = r^2$
radius $= 3.07$
diameter $= 2(3.07) = 6.14$ cm

Problems

Find the volume of each cylinder.

1. $r = 5$ cm, $h = 10$ cm
2. $r = 7.5$ in., $h = 8.1$ in.
3. diameter $= 10$ cm, $h = 5$ cm
4. base area $= 50 \text{ cm}^2$, $h = 4$ cm
5. $r = 17$ cm, $h = 10$ cm
6. $d = 29$ cm, $h = 13$ cm

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Find the missing part of each cylinder.

7. If the volume is 5175 ft^3 and the height is 23 ft, find the diameter.

8. If the volume is 26,101.07 $inches^3$ and the radius is 17.23 inches, find the height.

9. If the circumference is 126 cm and the height is 15 cm, find the volume.

Answers

1.	785.40 cm^3	2.	1431.39 $in.^3$	3.	392.70 cm^3
4.	200 cm^3	5.	9079.20 cm^3	6.	8586.76 cm^3
7.	16.93 ft	8.	28 inches	9.	18,950.58 cm^3

SURFACE AREA OF A CYLINDER

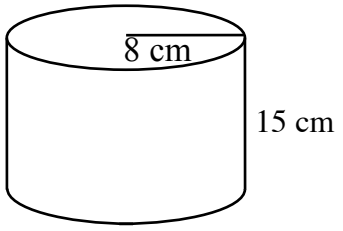
The surface area of a cylinder is the sum of the two base areas and the lateral surface area. The formula for the surface area is:

$$SA = 2r^2\pi + \pi dh \quad \text{or} \quad SA = 2r^2\pi + 2\pi rh$$

where r = radius, d = diameter, and h = height of the cylinder. For additional information, see the Math Notes box in Lesson 10.1.3 of the *Core Connections, Course 3* text.

Example 1

Find the surface area of the cylinder at right. Use a calculator for the value of π.

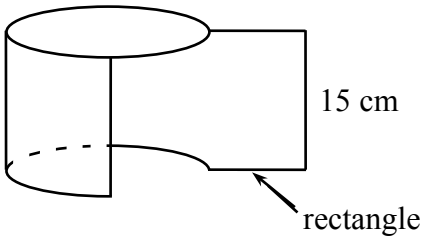


Step 1: Area of the two circular bases

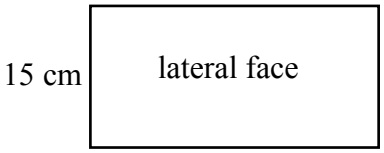
$$2[(8 \text{ cm})^2\pi] = 128\pi \text{ cm}^2$$

Step 2: Area of the lateral face

$$\pi(16)15 = 240\pi \text{ cm}^2$$



circumference of base = 16π cm



Step 3: Surface area of the cylinder

$$128\pi \text{ cm}^2 + 240\pi \text{ cm}^2 = 368\pi \text{ cm}^2 \approx 1156.11 \text{ cm}^2$$

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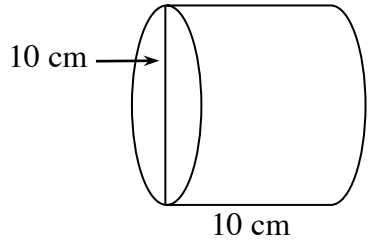
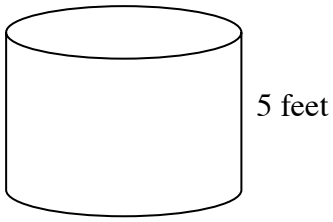
Example 2



$$\begin{aligned} SA &= 2r^2\pi + 2\pi rh \\ &= 2(5)^2\pi + 2\pi \cdot 5 \cdot 10 \\ &= 50\pi + 100\pi \\ &= 150\pi \approx 471.24 \text{ cm}^2 \end{aligned}$$

Example 3



If the volume of the tank above is 500π ft^3, what is the surface area?

$$\begin{aligned} V &= \pi r^2 h \\ 500\pi &= \pi r^2 (5) \\ \tfrac{500\pi}{5\pi} &= r^2 \\ 100 &= r^2 \\ 10 &= r \end{aligned}$$

$$\begin{aligned} SA &= 2r^2\pi + 2\pi rh \\ &= 2 \cdot 10^2\pi + 2\pi(10)(5) \\ &= 200\pi + 100\pi \\ &= 300\pi \approx 942.48 \text{ ft}^2 \end{aligned}$$

Problems

Find the surface area of each cylinder.

1. $r = 6$ cm, $h = 10$ cm
2. $r = 3.5$ in., $h = 25$ in.
3. $d = 9$ in., $h = 8.5$ in.
4. $d = 15$ cm, $h = 10$ cm
5. base area = 25, height = 8
6. volume = 1000 cm^3, height = 25 cm

Answers

1. 603.19 cm^2
2. 626.75 in.^{2}
3. 367.57 in.^{2}
4. 824.69 cm^2
5. 191.80 un.^{2}
6. 640.50 cm^2

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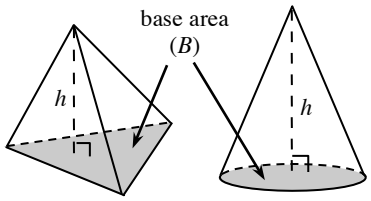
The volume of a pyramid is one-third the volume of the prism with the same base and height and the volume of a cone is one third the volume of the cylinder with the same base and height. The formula for the volume of the pyramid or cone with base B and height h is:



$$V = \frac{1}{3}Bh$$

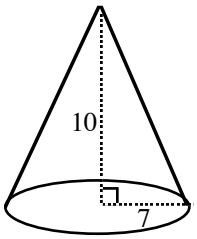
For the cone, since the base is a circle the formula may also be written:

$$V = \frac{1}{3}r^2\pi h$$

For additional information, see the Math Notes box in Lesson 10.1.4 of the *Core Connections, Course 3* text.

Example 1

Find the volume of the cone below.



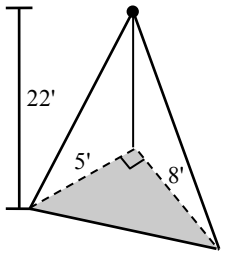
Volume $= \frac{1}{3}(7)^2\pi \cdot 10$

$= \frac{490\pi}{3}$

$\approx 513.13 \text{ units}^3$

Example 2

Find the volume of the pyramid below.



Base is a right triangle

$B = \frac{1}{2} \cdot 5 \cdot 8 = 20$

Volume $= \frac{1}{3} \cdot 20 \cdot 22$

$\approx 146.67 \text{ ft}^3$

Example 3

If the volume of a cone is 4325.87 cm^3 and its radius is 9 cm, find its height.

Volume $= \frac{1}{3}r^2\pi h$

$4325.87 = \frac{1}{3}(9)^2\pi \cdot h$

$12977.61 = \pi(81) \cdot h$

$\frac{12977.61}{81\pi} = h$

$51 \text{ cm} = h$

Problems

Find the volume of each cone.

1. $r = 4$ cm, $h = 10$ cm

2. $r = 2.5$ in, $h = 10.4$ in

3. $d = 12$ in, $h = 6$ in

4. $d = 9$ cm, $h = 10$ cm

5. $r = 6\frac{1}{3}$ ft, $h = 12\frac{1}{2}$ ft

6. $r = 3\frac{1}{4}$ ft, $h = 6$ ft

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Find the volume of each pyramid.

7. base is a square with side 8 cm
$h = 12$ cm

8. base is a right triangle with legs 4 ft and 6 ft
$h = 10\frac{1}{2}$ ft

9. base is a rectangle with width 6 in., length 8 in.
$h = 5$ in.

Find the missing part of each cone described below.

10. If V = 1000 cm^3 and $r = 10$ cm, find h.

11. If V = 2000 cm^3 and $h = 15$ cm, find r.

12. If the circumference of the base = 126 cm and $h = 10$ cm, find the volume.

Answers

1. 167.55 cm^3
2. 68.07 in^3
3. 226.19 in^3
4. 212.06 cm^3
5. 525.05 ft^3
6. 66.37 ft^3
7. 256 cm^3
8. 42 ft^3
9. 80 in^3
10. 9.54 cm
11. 11.28 cm
12. 4211.24 cm^3

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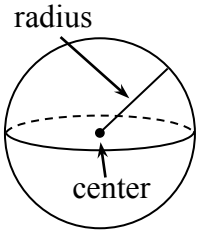
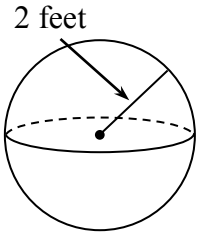
For a sphere with radius r, the volume is found using $V = \frac{4}{3}\pi r^3$.

For more information, see the Math Notes box in Lesson 10.1.5 of the *Core Connections, Course 3* text.



Example 1

Find the volume of the sphere at right.



$V = \frac{4}{3}\pi\ r^3 = \frac{4}{3}\pi\ 2^3 = \frac{32\pi}{3}\ \text{ft}^3$ exact answer

or using $\pi \approx 3.14$

$\frac{32(3.14)}{3} \approx 33.49\ \text{ft}^3$ approximate answer

Example 2

A sphere has a volume of 972π un.^{3}. Find the radius.

Use the formula for volume and solve the equation for the radius.

$V = \frac{4}{3}\pi r^3 = 972\pi$	Substitution.
$4\pi r^3 = 2916\pi$	Multiply by 3 to remove the fraction.
$r^3 = \frac{2916\pi}{4\pi} = 729$	Divide by 4π to isolate r.
$r = \sqrt[3]{729} = 9$	To undo cubing, take the cube root.

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Problems

Use the given information to find the exact and approximate volume of the sphere.

1. radius = 10 cm
2. radius = 4 ft
3. diameter = 10 cm
4. diameter = 3 miles
5. circumference of great circle = 12π un.
6. circumference of great circle = 3π un.

Use the given information to answer each question related to spheres.

7. If the radius is 7 cm, find the volume.
8. If the diameter is 10 inches, find the volume.
9. If the volume of the sphere is 36π un.^{3}, find the radius.
10. If the volume of the sphere is $\frac{256\pi}{3}$ un.^{3}, find the radius.

Answers

1. $\frac{4000\pi}{3} \approx 4186.67$ cm^3
2. $\frac{256\pi}{3} \approx 267.94$ ft^3
3. $\frac{500\pi}{3} \approx 523.33$ cm^3
4. $\frac{9\pi}{2} \approx 14.13$ mi^3
5. $288\pi \approx 904.32$ un.^{3}
6. $\frac{9\pi}{2} \approx 14.13$ un.^{3}
7. $\frac{1372\pi}{3} \approx 1436.75$ cm^3
8. $\frac{500\pi}{3} \approx 523.60$ in.^{3}
9. $r = 3$ units
10. $r = 4$ units

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